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Labor markets in heterogenous sectors

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Abstract

I expand the standard model with labor frictions and matching function, to account to the endogenous decision to either leave the labor market or migrate to a different sector, after a stochastic training period. The major empirical finding is that, after a monetary policy shock, the fall in employment, hours, real salaries, GDP and output is much faster and stronger in the manufacturing than in the services sector. The model is also able to capture what is known as labor hoarding, for hours tend to fall much faster than employment after the shock.

Keywords: DSGE, Sector Heterogeneity, Manufacturing and Services Sectors, Labor markets, Bayesian inference, Intensive and extensive margins of labor.

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1 Introduction

During the last 15 years the goods and labor markets in Brazil have been showing what Alves and Correa (2013) called the *Brazilian Labor Market Dichotomy*. The authors conjecture and find some evidence that this phenomenon was driven from deep sectoral heterogeneity between the manufacturing and services sectors.

Section 1.1 broadens the stylized facts related to this labor market dichotomy and presents evidence that the effects of sector heterogeneity have been more evident after the 2008-2009 Great Recession crisis. For instance, the unemployment rate have kept a decreasing path, even though activity measures were also led to fall: (a) the GDP growth rate; (b) the participation rate; and (c) important measures from the manufacturing sector only, such as GDP gap, employed workers, hours per worker and inflation rate.

One the one hand, those puzzling facts suggests that any analysis on production, labor market and inflation using Brazilian data must consider the strong heterogeneity of the services and manufacturing sectors, and must consider both the intensive and the extensive margins of labor in both sectors. On the other hand, increasing the sophistication of a general equilibrium model to account to such a level of heterogeneity might make inference much harder.

In this context, I aim at answering two important questions: (i) Which modelling features does a Dynamic General Equilibrium Model need in order to imbed the strong heterogeneity of the goods and labor markets in the services and manufacturing sectors, and account for those stylized facts observed in the Brazilian economy? (ii) How do sectoral labor and goods markets quantities respond to monetary policy shocks?

For answering the first question, I expand the standard DMP model[1] (after Diamond (1982), Mortensen (1982) and Pissarides (1985)), with search and matching frictions to account for equilibrium unemployment, to account to the endogenous decision to either leave the labor market or reallocate to a different sector, after a stochastic training period. Sectors (manufacturing and services) are asymmetric, firms are subject to sector-specific price stickiness and labor productivity, have specific labor force, post vacancies advertisement and explore both the intensive as the extensive margin of labor. For simplicity and better understanding the labor market interactions with the goods market, I consider a closed economy, with constant stock of capital and two sectors only: services and manufacturing.

In the labor market modelling part, I bring two important contributions. First, in order to account for an endogenous leave of the labor market, I assume that searching for a job is a burden, captured by a constant disutility per unemployed worker. This assumption is simple, but rich enough to capture the trade-off between searching for a job for an uncertain period of time, which brings unemployment compensations and the expectation of a salary in the future, and stop looking for a job for a while, which ends the frustration of unsuccessfully searching a job for a while, even though losing unemployment compensations. If was not for this

[1] While searching for jobs, unemployed workers earn monetary transfers and leisure benefits. Firms search for workers and post job vacancies at a cost. Search frictions prevents all unemployed workers from getting a job and firm from filling all available vacancies. Instead, the probability that an unemployed worker is matched into a new job depends on the total unemployed labor force and on the total number of vacancies. After a match occurs, individual wages are set by a Nash bargaining between the newly hired worker and the firm.

burden, unemployed workers with consumption insurance, as the ones that come back to their parents home or share a big household in which some of them have a job, would voluntarily prefer to remain unemployed. Indeed, they will consume just as an employed worker and have lower disutility to work.

In the literature, Christiano et al. (2010) uses a similar, but not as simple, way to account for endogenous involuntary unemployment. They assume that the disutility is a convex function of the time spent to search for a job, which in turns increases the chances of obtaining one. The way I model the burden, even though simpler, allows me for similar results.

Second, I model an asymmetric cost of reallocation to a different sector. Unemployed workers should leave the labor market, for a stochastic period of time, to specialize on the necessary skills for working in the other sector. When searching for a job in a different firm of the same sector, no specialization cost is imposed.

As in Thomas (2011) and Alves (2012), I assume that firms simultaneously make decisions on pricing and both the intensive and extensive margins of labor, so that labor is firm-specific. This interaction between pricing and firm-specific labor induces richer dynamics in both the goods and labor market.

Addressing the second question, I estimate log-linearized version of the model and analyse empirical responses to a monetary policy shock.

Estimation of 38 deep parameters and 13 standard deviations of the heterogeneous model is done using Bayesian technique with a Metropolis-Hasting MCMC algorithm and flat priors, except for two parameters capturing sectoral degrees of price rigidity,[2] using 13 observed quarterly variables, from 2003:Q1 to 2014:Q4: manufacturing (detrended) GDP, services (detrended) GDP, tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the manufacturing sector), non-tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the services sector), working-age population, participation rate, employed workers at the manufacturing sector, employed workers at the services sector, hours per worker at the manufacturing sector, aggregate hours per worker, separation rate at the manufacturing sector, total mass of hired workers, and nominal interest rate.

After convergence, which occurred after about 1,250,000 draws from the MCMC sampler, I keep the next 1,250,000 draws for inference and Bayesian impulse response exercises.

The major empirical findings are that: (i) workers from the manufacturing sector who are out of the labor market take longer to return (1.9 quarters) than workers from the service sector (1.1 quarters) ; (ii) workers from the manufacturing sector reallocate much faster to the service sector (2.3 quarters) than workers from the services sector - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare; (iii) it is the labor market tightness the major explanation why unemployed workers find it easier to get a job in the services sector than in the manufacturing one; (iv) although unemployed workers from the service sector find it easier to get a job than workers from the manufacturing sector, the workers' bargaining power in the manufacturing sector is much larger than the

[2] Reasons are detailed in Section 3.

bargaining power in the service sector. As a result, the average salary in the service sector are more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The results also suggest that salary bargaining is much more efficient in the manufacturing sector.

The data also support the evidence that there is no labor supply puzzle in the Brazilian labor market, i.e. I find that labor is just weakly elastic to salaries in Brazil.

The results also suggest that workers are much more productive, on average, in the manufacturing sector than those from the services sector. Moreover, prices are much stickier and much more persistent in the manufacturing sector than in the services sector. Since prices are more flexible in the services sector, its real side is not as much affected by monetary policy as it is in the manufacturing sector. And, even though price rigidity is stronger in the manufacturing sector, sectoral inflation dynamics must not detach as much due to strategic complementarity.

As for the dynamics after a monetary policy shock, the results imply that it is the manufacturing sector which suffers more. The fall in employment, hours, real salaries, GDP and output is much stronger in the manufacturing than in the services sector. The model is also able to capture what is know as labor hoarding, for hours tend to fall much faster than employment after the shock.

The remainder of his paper is organized as follows. Section 1.1 describe stylized facts of the goods and labor market in Brazil. Section 2 describes the model. Section 3 estimates the model, while Section 4 shows some impulse responses from selected shocks. Section 5 concludes.

1.1 Stylized facts

All variables described in this section were released by the Brazilian Institute of Geography and Statistics (IBGE). In particular, the labor market variables are obtained from the IBGE's Employment Monthly Survey (PME). At first glance, the most impressing fact is the ever-decreasing path of the unemployment rate, which was is not accompanied by increasing GDP growth rates. In fact, GDP was strongly hit by the 2008-2009 crisis, whereas the unemployment rate was barely affected at all, as depicted in panel A of figure 1. Alves and Correa (2013) state and find strongly evidence that this dichotomy is part of a big picture describing two different sectors in Brazil, i.e. manufacturing and services. For comparison, the services sector represents about 68% of Brazilian nominal GDP, as depicted in panel B of figure 1.Since the farming sector represents only 5.5% of Brazilian nominal GDP, and is strongly intensive in capital in Brazil, I embed it into the manufacturing GDP for the analysis I do in this paper.

As we look closer at sectoral specific data, we find that those two sectors are very heterogeneous in many dimensions. For instance, Panel C of figure 1 shows the GDP gaps of both sectors from their common (long-run) log-linear trend. Even obtained from different data, using a simpler method, this picture agrees with the findings of Alves and Correa (2013): the manufacturing GDP behave been struggling since 2011, after barely recovering from the (2008-2009) second Great Recession, while the services GDP was barely affected by the crisis. Only by mid-2014 this sector showed signs of struggle.

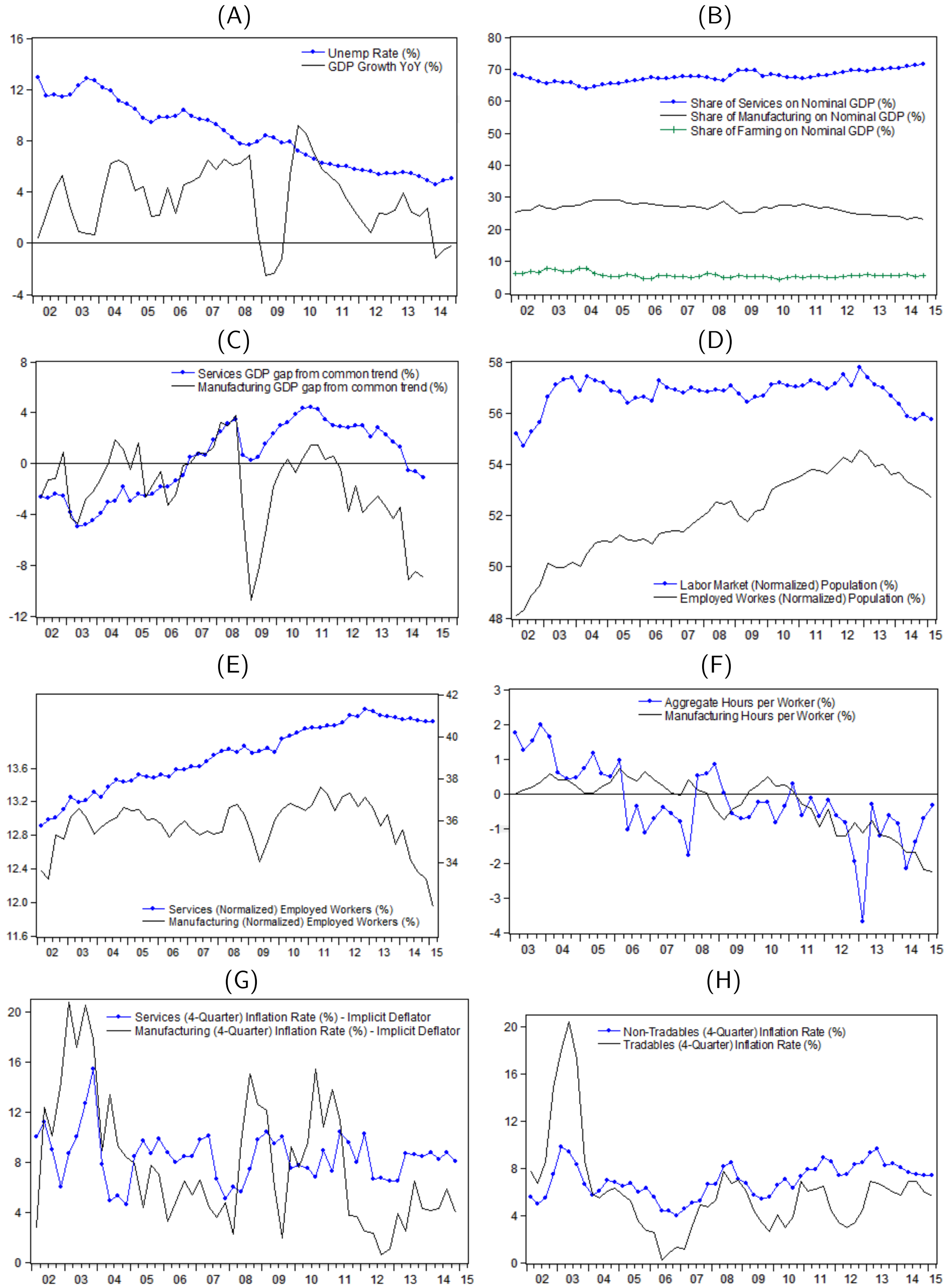

Figure 1: GDP, Labor Market and Inflation

As for the labor market, the annual growth rate of the active-age population has decreased from 1.7% in the early 2000's to about 1.2% by the 2010's, perhaps reflecting a demographic change towards an older population. Nevertheless, when normalizing by the active-age population, labor market stocks are more informative.

Panel D of figure 1 depicts the (normalized) labor market population, i.e. the participation rate, and (normalized) employed workers. Note that the labor market population remains stable for most of the sample, expect for falls at the beginning (2002) and the end of the sample (2013-2015), while the mass of employed workers has been steadily increasing until the end of 2012. At this point, both the participation rate and (normalized) employed workers started to decrease. Since the fall in the participation rate was larger than the fall of the latter, the unemployment continued to fall after 2012.

Panel E of figure 1 depicts the (normalized) masses of employed workers in the services and manufacturing sectors. Many features of the labor market suggest a strong heterogeneity. The first one is the fact that the services sector employs about 75% of the Brazilian working population. The remaining features come from their dynamics over time. Note that while the (normalized) employed population at the manufacturing sector remains stable for most of the sample, it has three periods of remarkable falls: (i) the beginning of the sample (2002); (ii) the second Great Recession (2008-2009); and (iii) the end of the sample (2013-2015). As for the services sector, its (normalized) employed population has been steadily increasing until the end of 2012, when its growth rate came to a halt. Note also that, differently of what happened in the manufacturing sector, the second Great Recession had almost no effect on the employed population of the services sector.

Panel E of figure 1 depicts actual hours per worker, both in the aggregate (PME) as in the manufacturing sector (PIMES), described in terms of percentage deviations from their sample averages. Note that hours per worker have important variability over the cycle and have different sectoral dynamics.

As for sectoral inflation rates, panel G of figure 1 shows the 4-Quarter inflation rates of the implicit deflators of the services and manufacturing GDPs. Note that, even though the inflation rate of the manufacturing sector is more volatile, its level is much lower that of the services sector. And the gap between them seemed have become even larger more after 2011. In order to compare with inflation rates observed by consumers, panel H shows the 4-Quarter inflation rates, from the Brazilian Broad Consumer Price Index (IPCA) of non-tradable and tradable goods, as also considered in Alves and Correa (2013). Note that the main message is the same, including the gap opening from 2012 to 2015, since most of non-tradable goods comes from the services sector and most of tradable goods comes from the manufacturing sector. As expected for consumption goods, the volatilities are smaller than that from the implicit deflators.

2 The model

The representative household consumes consumption goods and have a continuum of workers, which can be hired or lose their jobs. The labor market is subject to two sources of inefficiency: (i) workers can only work in their home economy; and (ii) there are search and match frictions. Finally, wages and hours are decided in a flexible Nash bargaining framework.

In each economy, consumption goods $z \in (0,1)$ can be either *manufactured* $(\mathfrak{m})$ or *services* $(\mathfrak{s})$ and are produced in two broad sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$, i.e. there is a $\mathfrak{w}_{\mathfrak{m}}$ mass of goods $z_{\mathfrak{m}} \in \mathcal{Z}_{\mathfrak{m}} \equiv (0, \bar{z}_{\mathfrak{m}}]$ from sector $\mathfrak{m}$, and a $\mathfrak{w}_{\mathfrak{s}} = 1 - \mathfrak{w}_{\mathfrak{m}}$ mass of goods $z_{\mathfrak{s}} \in \mathcal{Z}_{\mathfrak{s}} \equiv (\bar{z}_{\mathfrak{m}}, 1]$ from sector $\mathfrak{s}$. Whenever convenient, I use the notation z when the results are independent of the firm type.

In the producing industry, differentiated firms produce all sort of consumption goods. Firms use labor in both the extensive and the intensive margins, post job vacancies at a cost and make price decisions.

The model is depicted in Figure 2, which makes it easier to understand the whole structure in the analytical part.

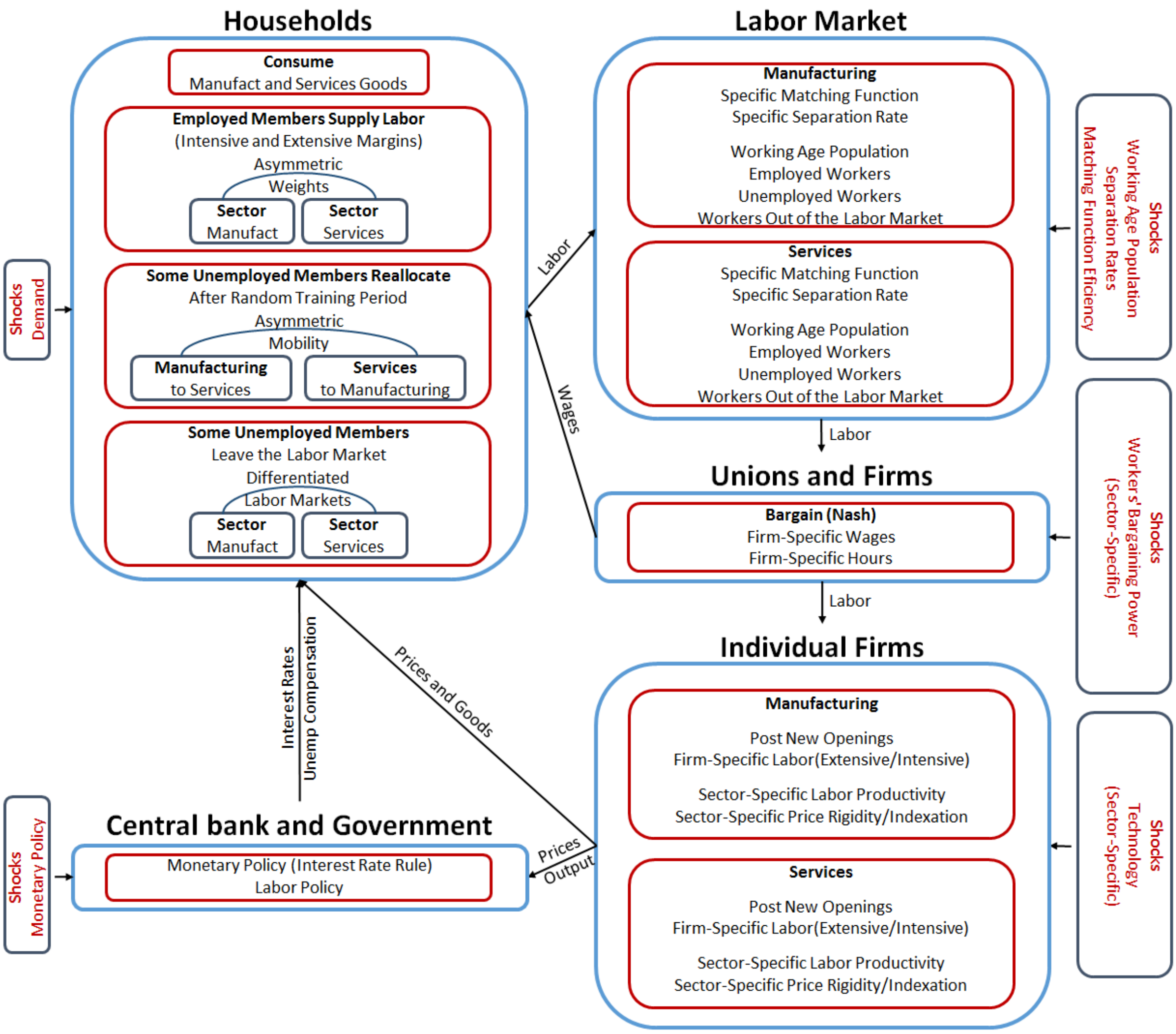

Figure 2: Model Structure

2.1 Labor flows

At the end of period t, the representative household has $\ell_t^{\mathfrak{p}}$ members at working age who care about all future generations. The size $\ell_t^{\mathfrak{p}}$ of the representative family is exogenous, stochastic, stationary, and its unconditional mean is normalized to unity, i.e. $E\ell_t^{\mathfrak{p}} = 1$.

Out of the $\ell_t^{\mathfrak{p}}$ members in the representative household, ℓ_t members are in the labor market (employed or unemployed) and $\ell_t^{\mathfrak{o}}$ members are out of the labor market. The quantities satisfy $\ell_t^{\mathfrak{p}} \equiv (\ell_t + \ell_t^{\mathfrak{o}})$. Even though the family size is an exogenous variable, the flows in and out the labor market are endogenously decided.

Within the household, $\mathsf{n}_t(z_{\mathfrak{c}}) \in (0, \ell_t)$ members are employed in firm $z_{\mathfrak{c}}$, from sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Labor is firm-specific and, due to labor market frictions, not all members are employed. In this context, $\mathsf{n}_t \equiv \int_0^1 \mathsf{n}_t(z)\, dz$ and $\mathsf{n}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{n}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ are the end-of-period employment aggregates in the economy as a whole and at sector $\mathfrak{c}$. During each period, $\mathsf{m}_t(z_{\mathfrak{c}})$ workers are matched into firm $z_{\mathfrak{c}}$. In this context, $\mathsf{m}_t \equiv \int_0^1 \mathsf{m}_t(z)\, dz \text{ and } \mathsf{m}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{m}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ are the aggregate new matches in the economy as a whole and at sector $\mathfrak{c}$. The definitions imply:

$$\mathsf{n}_t = \mathfrak{w}_{\mathfrak{m}} \mathsf{n}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{n}_{\mathfrak{s},t} \quad ; \; \mathsf{m}_t = \mathfrak{w}_{\mathfrak{m}} \mathsf{m}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{m}_{\mathfrak{s},t} \tag{1}$$

While unemployed, workers might get a job within their own sectors according to a matching technology, described in the end of this section, without bearing any extra cost.

After not being matched during each period in sector $\mathfrak{c}$, however, a mass $\mathsf{m}_{\mathfrak{c},t}^{\mathfrak{o}}$ of unemployed workers decide it is better not to search for a job for a while, and possibly decide it is time to reallocate to the other sector. In any case, those workers leave the labor force of sector $\mathfrak{c}$ and enroll at the specialization school of sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$, where she catches up with frontier skills needed for either returning the original sector or working in the other sector. Training is not easy, though. With probability $\delta_{\mathfrak{c}}^{\mathfrak{c}}$, each worker returns to the labor force of sector $\mathfrak{c}$ in the beginning of next period. With probability $\delta_{\mathfrak{c}}^{\bar{\mathfrak{c}}}$, she become fully specialized for working at sector $\bar{\mathfrak{c}} \neq \mathfrak{c}$ and decide it is better to reallocate to this sector in the beginning of next period.[3] In any case, specialized workers become part of the masses of beginning-of-period unemployed workers.

By the end of each period, $\mathsf{m}_t^{\mathfrak{o}}$ individuals have left the labor force, while $\ell_t^{\mathfrak{o}}$ aggregates all individuals out of the labor force:

$$\mathsf{m}_t^{\mathfrak{o}} \equiv \mathfrak{w}_{\mathfrak{m}} \mathsf{m}_{\mathfrak{m},t}^{\mathfrak{o}} + \mathfrak{w}_{\mathfrak{s}} \mathsf{m}_{\mathfrak{s},t}^{\mathfrak{o}} \tag{2}$$

$$\ell_t^{\mathfrak{o}} \equiv \mathfrak{w}_{\mathfrak{m}} \ell_{\mathfrak{m},t}^{\mathfrak{o}} + \mathfrak{w}_{\mathfrak{s}} \ell_{\mathfrak{s},t}^{\mathfrak{o}} \tag{3}$$

where $\ell_{\mathfrak{c},t}^{\mathfrak{o}}$ is the mass of individuals out of the labor force of each sector.

At the beginning of each period, employed members separate from their jobs at an exogenous time-varying rate $\rho_{\mathfrak{c}} \in (0,1)$, which I assume to evolve according to the following stationary process about its steady state level $\bar{\rho}_{\mathfrak{c}}$:

$$\frac{\rho_{\mathfrak{c},t}}{\bar{\rho}_{\mathfrak{c}}} = \epsilon_{\mathfrak{c},t}^{\rho} \left(\frac{\rho_{\mathfrak{c},t-1}}{\bar{\rho}_{\mathfrak{c}}} \right)^{\phi_{\mathfrak{c}}^{\rho}} \tag{4}$$

where $\epsilon_{\mathfrak{c},t}^{\rho}$ is the sector-$\mathfrak{c}$ specific shock on the separation rate and $\phi_{\mathfrak{c}}^{\rho} \in (0,1)$.

[3] Note that whenever $\delta_{\mathfrak{c}}^{\bar{\mathfrak{c}}} > \delta_{\bar{\mathfrak{c}}}^{\mathfrak{c}}$, it is easier to migrate from sector $\mathfrak{c}$ to sector $\bar{\mathfrak{c}}$ than from sector $\bar{\mathfrak{c}}$ to sector $\mathfrak{c}$.

Simultaneously, some individuals die and others come to working-age. I capture this fluctuation by assuming a constant exogenous death rate $\rho_{\mathfrak{d}} \in (0,1)$, affecting the masses of individuals in and out the labor market, and an exogenous net flow of $\mathsf{m}_{\ell,t}$ extra individuals evenly enrolling at specialization schools:

$$(\mathsf{m}_{\ell,t} - \bar{\mathsf{m}}_{\ell}) = \phi_{\ell} (\mathsf{m}_{\ell,t-1} - \bar{\mathsf{m}}_{\ell}) + \epsilon_{\ell,t} \tag{5}$$

where $\bar{\mathsf{m}}_{\ell}$ is the steady state level of extra individuals coming to working-age, $\epsilon_{\ell,t}$ is a shock to the mass of individuals coming to working-age and $\phi_{\ell} \in (0,1)$. and $\phi_{\ell} \in (0,1)$.

Based on those features, the laws of motion of employed members are described by

$$\begin{aligned} \mathsf{n}_t (z_{\mathfrak{c}}) &= (1-\rho_{\mathfrak{d}}) \left(1-\rho_{\mathfrak{c},t-1}\right) \mathsf{n}_{t-1} (z_{\mathfrak{c}}) + \mathsf{m}_t (z_{\mathfrak{c}}) \\ \mathsf{n}_{\mathfrak{c},t} &= (1-\rho_{\mathfrak{d}}) \left(1-\rho_{\mathfrak{c},t-1}\right) \mathsf{n}_{\mathfrak{c},t-1} + \mathsf{m}_{\mathfrak{c},t} \end{aligned} \tag{6}$$

The sectoral masses of individuals out of the labor market evolve as follows:

$$\ell^{\mathfrak{o}}_{\mathfrak{c},t} = (1-\rho_{\mathfrak{d}}) \, \ell^{\mathfrak{o}}_{\mathfrak{c},t-1} - \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} - \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} + \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t} \tag{7}$$

where $\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t}$ and $\mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t}$ denotes the flow of workers out of the labor force of sector $\mathfrak{c}$ who either returns to sector $\mathfrak{c}$ to search for a job or reallocates to sector $\bar{\mathfrak{c}}$, $\mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t}$ is the flow of workers coming from out of the labor market into sector $\mathfrak{c}$, and $\mathsf{m}_{\mathfrak{o},t}$ is the total flow of workers coming from out of the labor market. Those masses are defined as follows:

$$\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} = \delta^{\mathfrak{c}}_{\mathfrak{c}} (1-\rho_{\mathfrak{d}}) \, \ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \quad ; \; \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} = \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} (1-\rho_{\mathfrak{d}}) \, \ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \tag{8}$$

$$\mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} \equiv \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}} \mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t} \quad ; \; \mathsf{m}_{\mathfrak{o},t} \equiv \mathfrak{w}_{\mathfrak{m}} \mathsf{m}^{\mathfrak{m}}_{\mathfrak{o},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{m}^{\mathfrak{s}}_{\mathfrak{o},t} \tag{9}$$

The beginning-of-period unemployment aggregates u_t and $u_{\mathfrak{c},t}$ account for unemployed members at the end of last period u^e_{t-1} and $\mathsf{u}^e_{\mathfrak{c},t-1}$ (defined further on), added to recently separated workers and workers returning from out of the labor market. Because I use a quarterly frequency calibration, I follow Ravenna and Walsh (2010) in distinguishing beginning-of-period from end-of-period unemployment aggregates. This strategy accounts for time-aggregation issues. The laws of motion are the following:

$$\begin{aligned} \mathsf{u}_{\mathfrak{c},t} &= (1-\rho_{\mathfrak{d}}) \left(\mathsf{u}^e_{\mathfrak{c},t-1} + \rho_{\mathfrak{c},t-1} \mathsf{n}_{\mathfrak{c},t-1}\right) + \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} \\ \mathsf{u}_t &= \mathfrak{w}_{\mathfrak{m}} \mathsf{u}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{u}_{\mathfrak{s},t} \end{aligned} \tag{10}$$

Considering the masses $\mathsf{m}_{\mathfrak{c},t}$ and $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$ of unemployed workers either matched into a new job in sector c or leaving the labor force, the end-of-period unemployment aggregates u^e_t and $\mathsf{u}^e_{\mathfrak{c},t}$ and labor forces ℓ_t and $\ell_{\mathfrak{c},t}$

and are defined as follows:

$$\mathsf{u}^{e}_{\mathfrak{c},t} = \mathsf{u}_{\mathfrak{c},t} - \mathsf{m}_{\mathfrak{c},t} - \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} \quad ; \; \mathsf{u}^{e}_{t} = \mathfrak{w}_{\mathfrak{m}} \mathsf{u}^{e}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{u}^{e}_{\mathfrak{s},t} \tag{11}$$

$$\ell_{\mathfrak{c},t} = \mathsf{u}^{e}_{\mathfrak{c},t} + \mathsf{n}_{\mathfrak{c},t} \quad ; \; \ell_{t} = \mathfrak{w}_{\mathfrak{m}} \ell_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \ell_{\mathfrak{s},t} \tag{12}$$

Total and sectoral working-age populations are defined as follows:

$$\ell^{\mathfrak{p}}_{\mathfrak{c},t} \equiv \ell_{\mathfrak{c},t} + \ell^{\mathfrak{o}}_{\mathfrak{c},t} \quad ; \; \ell^{\mathfrak{p}}_{t} \equiv \mathfrak{w}_{\mathfrak{m}} \ell^{\mathfrak{p}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \ell^{\mathfrak{p}}_{\mathfrak{s},t} \tag{13}$$

From (12),(11), (10), (6) and (9), we obtain alternative laws of motion for $\ell_{\mathfrak{c},t}$ and ℓ_{t}:

$$\ell_{\mathfrak{c},t} = (1 - \rho_{\mathfrak{d}})\, \ell_{\mathfrak{c},t-1} + \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} - \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} \quad \ell_{t} = (1 - \rho_{\mathfrak{d}})\, \ell_{t-1} + \mathsf{m}_{\mathfrak{o},t} - \mathsf{m}^{\mathfrak{o}}_{t} \tag{14}$$

From (13), (7), (14), and (9), we obtain alternative laws of motion for $\ell^{\mathfrak{p}}_{\mathfrak{c},t}$ and $\ell^{\mathfrak{p}}_{t}$:

$$\ell^{\mathfrak{p}}_{\mathfrak{c},t} = (1 - \rho_{\mathfrak{d}})\, \ell^{\mathfrak{p}}_{\mathfrak{c},t-1} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}} \mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t} - \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t} \quad ; \; \ell^{\mathfrak{p}}_{t} = (1 - \rho_{\mathfrak{d}})\, \ell^{\mathfrak{p}}_{t-1} + \mathsf{m}_{\ell,t} \tag{15}$$

which implies that the working-age population $\ell^{\mathfrak{p}}_{t}$ evolves according to a completely exogenous AR(1) process. Note that the condition $E\ell^{\mathfrak{p}}_{t} = 1$ implies that $E\mathsf{m}_{\ell,t} = \bar{\mathsf{m}}_{\ell} = \rho_{\mathfrak{d}}$.

Standard end-of-period unemployment rates $\mathfrak{u}^{e}_{t}$ and $\mathfrak{u}^{e}_{\mathfrak{c},t}$ are defined as $\mathfrak{u}^{e}_{t} \equiv \frac{\mathsf{u}^{e}_{t}}{\ell_{t}}$ and $\mathfrak{u}^{e}_{\mathfrak{c},t} \equiv \frac{\mathsf{u}^{e}_{\mathfrak{c},t}}{\ell_{\mathfrak{c},t}}$, while participation rates $\mathfrak{r}_{t}$ and $\mathfrak{r}_{\mathfrak{c},t}$ are defined according to $\mathfrak{r}_{t} \equiv \frac{\ell_{t}}{\ell^{\mathfrak{p}}_{t}}$ and $\mathfrak{r}_{\mathfrak{c},t} \equiv \frac{\ell_{\mathfrak{c},t}}{\ell^{\mathfrak{p}}_{\mathfrak{c},t}}$.

Firm z posts $\mathsf{v}^{e}_{t}(z)$ job vacancies at the end of each period, and hence $\mathsf{v}_{t}(z) \equiv \mathsf{v}^{e}_{t-1}(z)$ is the mass of job openings at firm z available at the beginning of period t. Therefore, I define $\mathsf{v}^{e}_{t} \equiv \int_{0}^{1} \mathsf{v}^{e}_{t}(z)\, dz$ and $\mathsf{v}^{e}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{v}^{e}_{t}(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ as the total end-of-period number of vacancy postings in the economy as a whole and in sector $\mathfrak{c}$. Similarly, I define v_{t} and $\mathsf{v}_{\mathfrak{c},t}$ as the corresponding beginning-of-period job openings. Those quantities satisfy $\mathsf{v}_{t} = \mathsf{v}^{e}_{t-1}$, $\mathsf{v}_{\mathfrak{c},t} = \mathsf{v}^{e}_{\mathfrak{m},t-1}$, $\mathsf{v}_{t} = \mathfrak{w}_{\mathfrak{m}} \mathsf{v}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{v}_{\mathfrak{s},t}$ and $\mathsf{v}^{e}_{t} = \mathfrak{w}_{\mathfrak{m}} \mathsf{v}^{e}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{v}^{e}_{\mathfrak{s},t}$.

In this context, p_{t}, q_{t} and θ_{t} are the economy wide job-finding rate, matching rate and labor market tightness rate within the period. Those rates, and their corresponding sectoral peers, are defined as follows:

$$\begin{array}{llllll} \mathsf{p}_{t} \equiv \frac{\mathsf{m}_{t}}{\mathsf{u}_{t}} & ; \; \mathsf{p}_{\mathfrak{c},t} \equiv \frac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{u}_{\mathfrak{c},t}} & \mathsf{q}_{t} \equiv \frac{\mathsf{m}_{t}}{\mathsf{v}_{t}} & ; \; \mathsf{q}_{\mathfrak{c},t} \equiv \frac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{v}_{\mathfrak{c},t}} & \theta_{t} \equiv \frac{\mathsf{v}_{t}}{\mathsf{u}_{t}} & ; \; \theta_{\mathfrak{c},t} \equiv \frac{\mathsf{v}_{\mathfrak{c},t}}{\mathsf{u}_{\mathfrak{c},t}} \\ \mathsf{p}^{e}_{t} \equiv \frac{\mathsf{m}_{t}}{\mathsf{u}^{e}_{t}} & ; \; \mathsf{p}^{e}_{\mathfrak{c},t} \equiv \frac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{u}^{e}_{\mathfrak{c},t}} & \mathsf{q}^{e}_{t} \equiv \frac{\mathsf{m}_{t}}{\mathsf{v}^{e}_{t}} & ; \; \mathsf{q}^{e}_{\mathfrak{c},t} \equiv \frac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{v}^{e}_{\mathfrak{c},t}} & \theta^{e}_{t} \equiv \frac{\mathsf{v}^{e}_{t}}{\mathsf{u}^{e}_{t}} & ; \; \theta^{e}_{\mathfrak{c},t} \equiv \frac{\mathsf{v}^{e}_{\mathfrak{c},t}}{\mathsf{u}^{e}_{\mathfrak{c},t}} \end{array} \tag{16}$$

The sectoral matching functions have standard Cobb-Douglas forms,[4] i.e. $\mathsf{m}_{\mathfrak{c},t} \equiv \eta_{\mathfrak{c},t} \mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t} \mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t}$, where $a_{\mathfrak{c}} \in (0,1)$ and $\eta_{\mathfrak{c},t}$ measures the efficiency of the matching technology of sector $\mathfrak{c}$, which evolves according to the following stationary process about its steady state level, i.e. $\frac{\eta_{\mathfrak{c},t}}{\bar{\eta}_{\mathfrak{c}}} = \epsilon^{\eta}_{\mathfrak{c},t} \left(\frac{\eta_{\mathfrak{c},t-1}}{\bar{\eta}_{\mathfrak{c}}} \right)^{\phi^{\eta}_{\mathfrak{c}}}$, where $\epsilon^{\eta}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ specific shock on the efficiency of the matching technology and $\phi^{\rho}_{\mathfrak{c}} \in (0,1)$.

[4] In the literature of search frictions in the labor market, the standard form is Cobb-Douglas (e.g. Shimer (2005) and Pissarides (2000)).

All previous relations imply the following identity: $\mathsf{p}_t = \mathfrak{w}_{\mathfrak{m}} \frac{\mathsf{u}_{\mathfrak{m},t}}{\mathsf{u}_t} \mathsf{p}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \frac{\mathsf{u}_{\mathfrak{s},t}}{\mathsf{u}_t} \mathsf{p}_{\mathfrak{s},t}$. The intuition for this result is that the economy wide job-finding rate p_t can be computed using conditional probabilities. The conditional probability that an unemployed worker, at the beginning of period t, finds a job during during the period, given that she was in sector $\mathfrak{p}$ is $\mathsf{p}_{\mathfrak{c},t}$. Recall now that $\mathsf{u}_t = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t}$. It implies that the probabilities that an unemployed worker is either from sector $\mathfrak{m}$ or $\mathfrak{s}$, at the beginning of period t, are just the masses ratios:

$$\mathfrak{p}^{\mathsf{u}}_{\mathfrak{c},t} \equiv \frac{\mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t}}{\mathsf{u}_t} \tag{17}$$

As long as matching functions depend only on sectoral rates, such as unemployment and vacancies masses, firms are unable to influence the sectoral matching rate $\mathsf{q}_{\mathfrak{c},t}$. Firms and unions know this result, but do not internalize the specific form of the aggregate matching function. Therefore, the individual matching functions satisfy $\mathsf{m}_t(z_{\mathfrak{c}}) = \mathsf{q}_{\mathfrak{c},t} \mathsf{v}_t(z_{\mathfrak{c}})$. In the context of asymmetric sectors, I define the aggregate and sectoral rates $\mathsf{p}^{\mathsf{o}}_t$ and $\mathsf{p}^{\mathsf{o}}_{\mathfrak{c},t}$ according to which unemployed workers decide to leave the labor market: $\mathsf{p}^{\mathsf{o}}_t \equiv \frac{\mathsf{m}^{\mathsf{o}}_t}{(1-\mathsf{p}_t)\mathsf{u}_t}$ and $\mathsf{p}^{\mathsf{o}}_{\mathfrak{c},t} \equiv \frac{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}}{(1-\mathsf{p}_{\mathfrak{c},t})\mathsf{u}_{\mathfrak{c},t}}$, where $(1-\mathsf{p}_t)\,\mathsf{u}_t$ and $(1-\mathsf{p}_{\mathfrak{c},t})\,\mathsf{u}_{\mathfrak{c},t}$ are the masses of aggregate and sectoral unemployed workers who are not matched into new jobs during the period.

All previous relations imply the following alternative definitions for u^e_t and $\mathsf{u}^e_{\mathfrak{c},t}$: $\mathsf{u}^e_t = (1-\mathsf{p}^{\mathsf{o}}_t)(1-\mathsf{p}_t)\,\mathsf{u}_t$ and $\mathsf{u}^e_{\mathfrak{c},t} = \left(1-\mathsf{p}^{\mathsf{o}}_{\mathfrak{c},t}\right)(1-\mathsf{p}_{\mathfrak{c},t})\,\mathsf{u}_{\mathfrak{c},t}$.

For an unemployed worker at the beginning of period t in sector c , the expected spell $\mathsf{T}^u_{\mathfrak{c},t}$ until being matched into a job (in any sector) evolves according to:

$$\begin{aligned} \mathsf{T}^u_{\mathfrak{c},t} = \quad & \mathsf{p}_{\mathfrak{c},t}\bar{\mathsf{t}} + (1-\mathsf{p}_{\mathfrak{c},t})\left[1 + \left(1-\mathsf{p}^{\mathsf{o}}_{\mathfrak{c},t}\right) E_t \mathsf{T}^u_{\mathfrak{c},t+1}\right] \\ + \quad & (1-\mathsf{p}_{\mathfrak{c},t})\left[\mathsf{p}^{\mathsf{o}}_{\mathfrak{c},t}\left(\delta^{\bar{\mathfrak{c}}}_{\mathfrak{s}} E_t \mathsf{T}^u_{\bar{\mathfrak{c}},t+1} + \delta^{\mathfrak{c}}_{\mathfrak{c}} E_t \mathsf{T}^u_{\mathfrak{c},t+1} + \frac{1-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} - \delta^{\mathfrak{c}}_{\mathfrak{c}}}{\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} + \delta^{\mathfrak{c}}_{\mathfrak{c}}}\right)\right] \end{aligned} \tag{18}$$

where $\bar{t} \in (0,1)$ is the average time within a period in which a recently laid-off worker remains unemployed when he is matched to a new job by the end of the same period. Therefore, the expected spell T^u_t until being matched into a job, independently of the sector status, evolves according to:

$$\mathsf{T}^u_t = \mathfrak{p}^{\mathsf{u}}_{\mathfrak{m},t} \mathsf{T}^u_{\mathfrak{m},t} + \mathfrak{p}^{\mathsf{u}}_{\mathfrak{s},t} \mathsf{T}^u_{\mathfrak{s},t} \tag{19}$$

2.2 Domestic households

Besides making optimal consumption allocation, as described further on, the representative household is specialized in producing sectoral consumption bundles for own consumption and to be sold to firms as intermediate goods for posting vacancies. This market is competitive and hence the household makes zero profit out of it.

2.2.1 Consumption bundles

Consumption bundles are defined in terms of the economy wide consumption of goods from sectors $c \in F_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Households are in charge to produce $\mathfrak{C}_{\mathfrak{c},t}$ units of sectoral consumption bundles to be consumed by different agents in the economy wide. For that, the household needs to buy goods from domestic firms, i.e. $c_t(z_{\mathfrak{c}})$ units of manufactured good $z_{\mathfrak{c}}$, and use the following Dixit and Stiglitz (1977) CES technologies:

$$(\mathfrak{C}_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}} = \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\right)^{\frac{1}{\phi}} \int_{\mathfrak{c}} \boldsymbol{c}_t(z_{\mathfrak{c}})^{\frac{\phi-1}{\phi}} dz_{\mathfrak{c}}$$

at total cost $P_{\mathfrak{c},t}\mathfrak{C}_{\mathfrak{c},t} \equiv \int_{\mathfrak{c}} p_t(z_{\mathfrak{c}})\, \boldsymbol{c}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$, where $P_{\mathfrak{c},t}$ is the aggregate price of the sectoral bundle $\mathfrak{C}_{\mathfrak{c},t}$, and $\phi > 1$ is the elasticity of substitution between goods in the same sector.

The representative household consumes $C_{\mathfrak{c},t}$ units of the sectoral consumption bundle $\mathfrak{C}_{\mathfrak{c},t}$ and has utility over the aggregate consumption C_t, defined according to the CES technology $(C_t)^{\frac{\phi-1}{\phi}} = \sum_{\mathfrak{c}} (\mathfrak{w}_{\mathfrak{c}})^{\frac{1}{\phi}} (C_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}}$, at total cost $P_t C_t \equiv \sum_{\mathfrak{c}} P_{\mathfrak{c},t} C_{\mathfrak{c},t}$, where P_t is the aggregate consumer price index.

Generalizing Ravenna and Walsh (2010), I assume that each firm $z_{\mathfrak{c}}$ needs to buy $\mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{c}})$ and $\mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{c}})$ units of sectoral consumption bundles from sectors $\mathfrak{m}$ and $\mathfrak{s}$ in order to post $\mathsf{v}_t^e(z_{\mathfrak{c}})$ units of end-of-period job vacancies, according to the CES technology $(\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}} = (\mathfrak{w}_{\mathfrak{m}})^{\frac{1}{\phi}} (\mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}} + (\mathfrak{w}_{\mathfrak{s}})^{\frac{1}{\phi}} (\mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}}$, at total cost $P_t \mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) \equiv P_{\mathfrak{m},t}\mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{m}}) + P_{\mathfrak{s},t}\mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{s}})$, where $\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}})$ is proportional to the firm's end-of-period posted vacancies $\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) \equiv \varsigma_{\mathsf{v}\mathfrak{c}} \mathsf{v}_t^e(z_{\mathfrak{c}})$ and $\varsigma_{\mathsf{v}\mathfrak{c}}$ is a sector-$\mathfrak{c}$ specific proportionality parameter.

In order to simplify the notation, let $\wp_{\mathrm{j},t}$ denote the relative price of the sectoral consumption bundle with price $P_{\mathrm{j},t}$ with respect to the consumption aggregate price P_t: $\wp_{\mathrm{j},t} \equiv \frac{P_{\mathrm{j},t}}{P_t}$ and $\wp_{\mathrm{j},t} = \wp_{\mathrm{j},t-1}\frac{\Pi_{\mathrm{j},t}}{\Pi_t}$.

Expenditure minimization implies the following relations:

$$\begin{array}{ll} (\mathfrak{C}_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}} = \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\right)^{\frac{1}{\phi}} \int_{\mathfrak{c}} \boldsymbol{c}_t(z_{\mathfrak{c}})^{\frac{\phi-1}{\phi}} dz_{\mathfrak{c}} & ;\ P_{\mathfrak{c},t}^{1-\phi} = \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} p_t(zc)^{1-\phi} dz_{\mathfrak{c}} \\ \boldsymbol{c}_t(z_{\mathfrak{c}}) = \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathfrak{C}_{\mathfrak{c},t} \left(\frac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi} & ;\ P_{\mathfrak{c},t}\mathfrak{C}_{\mathfrak{c},t} = \int_{\mathfrak{c}} p_t(z_{\mathfrak{c}})\, \boldsymbol{c}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}} \end{array} \tag{20}$$

$$\begin{array}{ll} (C_t)^{\frac{\phi-1}{\phi}} = \sum_{\mathfrak{c}} (\mathfrak{w}_{\mathfrak{c}})^{\frac{1}{\phi}} (C_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}} & ;\ 1 = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} (\wp_{\mathfrak{c},t})^{1-\phi} \\ C_{\mathfrak{c},t} = \mathfrak{w}_{\mathfrak{c}} C_t (\wp_{\mathfrak{c},t})^{-\phi} & ;\ C_t = \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t} C_{\mathfrak{c},t} \end{array} \tag{21}$$

$$\begin{array}{c} \mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{c}}) = \mathfrak{w}_{\mathfrak{m}} \mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) (\wp_{\mathfrak{m},t})^{-\phi} \quad ;\ \mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{c}}) = \mathfrak{w}_{\mathfrak{s}} \mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) (\wp_{\mathfrak{s},t})^{-\phi} \\ \mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) = \wp_{\mathfrak{m},t}\mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{c}}) + \wp_{\mathfrak{s},t}\mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{c}}) \end{array} \tag{22}$$

Let $\mathsf{c}_{\mathfrak{c},t}^{\mathsf{vm}} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{c}_t^{\mathsf{vm}}(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ and $\mathsf{c}_{\mathfrak{c},t}^{\mathsf{vs}} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{c}_t^{\mathsf{vs}}(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ denote aggregate consumptions of goods from sectors m and s used as intermediates for posting vacancies by firms in production sector $\mathfrak{c}$. Those aggregates imply the following relations:

$$\mathsf{c}_{\mathfrak{m},t}^{\mathsf{v}\mathfrak{c}} \equiv \mathfrak{w}_{\mathfrak{c}} \varsigma_{\mathsf{vm}} \mathsf{v}_{\mathfrak{m},t}^e (\wp_{\mathfrak{c},t})^{-\phi} \quad ;\ \mathsf{c}_{\mathfrak{s},t}^{\mathsf{v}\mathfrak{c}} \equiv \mathfrak{w}_{\mathfrak{c}} \varsigma_{\mathsf{vs}} \mathsf{v}_{\mathfrak{s},t}^e (\wp_{\mathfrak{c},t})^{-\phi} \tag{23}$$

Since the household supplies consumption goods at zero profit, the household itself and firms buy $C_{\mathfrak{c},t}$, $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{m},t}$ and $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{s},t}$ units of the economy wide consumption bundle type $\mathfrak{c}$ at aggregate price $P_{\mathfrak{c},t}$. Equilibrium requires:

$$\mathfrak{C}_{\mathfrak{c},t} = C_{\mathfrak{c},t} + \mathfrak{w}_{\mathfrak{m}}\mathsf{c}^{\mathsf{vc}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\mathsf{c}^{\mathsf{vc}}_{\mathfrak{s},t} = \mathfrak{w}_{\mathfrak{c}}\mathfrak{C}_t\left(\wp_{\mathfrak{c},t}\right)^{-\phi} \tag{24}$$

where $\mathfrak{C}_t = \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t}\mathfrak{C}_{\mathfrak{c},t}$ is the aggregate expenditure over all consumption sectors $c \in F_{\mathfrak{c}}$. Using the previous results, I obtain

$$\mathfrak{C}_t = C_t + \mathfrak{w}_{\mathfrak{m}}\varsigma_{\mathsf{vm}}\mathsf{v}^{e}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\varsigma_{\mathsf{vs}}\mathsf{v}^{e}_{\mathfrak{s},t} \tag{25}$$

Note that $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{m},t}$ and $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{s},t}$ are to be interpreted as firms' intermediate consumption, which should be netted out when computing the model's GDP. See Section 2.3 for more details.

2.2.2 Optimal consumption allocation

As in Merz (1995), I assume full risk sharing of consumption among household members, employed, unemployed and out of the labor market.[5] All $\ell^{\mathfrak{p}}_t$ household members pool their income, and hence the household consumes $C_{\mathfrak{c},t}$ units of each type-$\mathfrak{c}$ consumption bundle. Unemployed workers earn monetary transfers from the government until they are matched into a firm. That generates $\mathfrak{w}_{\mathfrak{m}}P_t\varpi^{c}_{\mathfrak{m},t}\mathsf{u}^{e}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}P_t\varpi^{c}_{\mathfrak{s},t}\mathsf{u}^{e}_{\mathfrak{s},t}$ in nominal income for the household, where $\varpi^{c}_{\mathfrak{m},t}$ and $\varpi^{c}_{\mathfrak{s},t}$ are sectoral aggregate real unemployment compensations, which evolve according to the following exogenous processes:

$$\begin{aligned} &\varpi^{c}_{\mathfrak{m},t} = \epsilon_{\varpi,t}\left(\varpi^{c}_{\mathfrak{m},t-1}\right)^{\phi_\varpi}\left(\gamma^{c}_{\mathfrak{m}}\tilde{\varpi}_{\mathfrak{m},t-1}\right)^{1-\phi_\varpi} \quad ; \; \varpi^{c}_{\mathfrak{s},t} = \epsilon_{\varpi,t}\left(\varpi^{c}_{\mathfrak{s},t-1}\right)^{\phi_\varpi}\left(\gamma^{c}_{\mathfrak{s}}\tilde{\varpi}_{\mathfrak{s},t-1}\right)^{1-\phi_\varpi} \\ &\tilde{\varpi}_{\mathfrak{m},t} \equiv \left(\bar{\varpi}_{\mathfrak{m}}\right)^{\phi^{ss}_\varpi}\left(\varpi_{\mathfrak{m},t}\right)^{1-\phi^{ss}_\varpi} \quad ; \; \tilde{\varpi}_{\mathfrak{s},t} \equiv \left(\bar{\varpi}_{\mathfrak{s}}\right)^{\phi^{ss}_\varpi}\left(\varpi_{\mathfrak{s},t}\right)^{1-\phi^{ss}_\varpi} \end{aligned} \tag{26}$$

where $\epsilon_{\varpi,t}$ is an aggregate shock to unemployment compensation, $\varpi_{\mathfrak{c},t}$ is the aggregate salary at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$, $\bar{\varpi}_{\mathfrak{c}}$ is the steady state level of the aggregate salary, $\gamma^{c}_{\mathfrak{c}}$ is the steady state fraction of the aggregate salary given as unemployment compensation, and $\phi_\varpi \in (0,1)$.

The economy wide aggregate real unemployment compensation ϖ^{c}_t is defined as $\varpi^{c}_t = \frac{1}{\mathsf{u}^{e}_t}\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\varpi^{c}_{\mathfrak{c},t}\mathsf{u}^{e}_{\mathfrak{c},t}$.

Consumption over consumption bundle C_t provides utility[6] $u_t \equiv \mathfrak{u}_{\mathfrak{u},t}\frac{\left(C_t - \iota_{\mathfrak{u}}\tilde{C}_{t-1}\right)^{1-\sigma}}{(1-\sigma)}$ for each household member, where $\tilde{C}_t$ is the average consumption level which equals C_t in equilibrium, σ is the reciprocal of the intertemporal rate of substitution, $\iota_{\mathfrak{u}} \in (0,1)$ is the external habit formation parameter, and $\mathfrak{u}_{\mathfrak{u},t}$ is a preference shock, which evolves according to $\frac{\mathfrak{u}_{\mathfrak{u},t}}{\bar{\mathfrak{u}}_{\mathfrak{u}}} = \epsilon_{\mathfrak{u},t}\left(\frac{\mathfrak{u}_{\mathfrak{u},t-1}}{\bar{\mathfrak{u}}_{\mathfrak{u}}}\right)^{\phi_{\mathfrak{u}}}$, where $\epsilon_{\mathfrak{u},t}$ is the preference innovation and $\phi_{\mathfrak{u}} \in (0,1)$.

As in Alves (2012), the representative household has unions specialized in negotiating wage and hours with firms. Union $z_{\mathfrak{s}}$ represents all $\mathsf{n}_t(z_{\mathfrak{c}})$ workers when bargaining with firm $z_{\mathfrak{s}}$ on hours per worker $h_t(z_{\mathfrak{c}})$ and nominal hourly wages $W_t(z_{\mathfrak{c}}) = P_t w_t(z_{\mathfrak{c}})$, where $w_t(z_{\mathfrak{c}})$ is the real wage. Whenever convenient, I

[5] Some authors have been making efforts to model imperfect consumption insurance and fully capture the distortions caused by unemployment. See e.g. Christiano et al. (2010).

[6] See e.g. Abel (1990) and Gali (1994).

consider instead the total nominal and real salaries over the period $\mathcal{W}_t(z_{\mathfrak{c}}) = W_t(z_{\mathfrak{c}})\, h_t(z_{\mathfrak{c}})$ and $\varpi_t(z_{\mathfrak{c}}) = w_t(z_{\mathfrak{c}})\, h_t(z_{\mathfrak{c}})$. Total hours worked at firm $z_{\mathfrak{s}}$ is defined as $H_t(z_{\mathfrak{c}}) \equiv \mathsf{n}_t(z_{\mathfrak{c}})\, h_t(z_{\mathfrak{c}})$.

Representing the workers, the union's disutility to $H_t(z_{\mathfrak{c}})$ is $\upsilon_t(z_{\mathfrak{c}}) \equiv \chi \frac{H_t(z_{\mathfrak{c}})^{1+\nu}}{(1+\nu)}$, where ν is the reciprocal of Frisch labor elasticity. Since the unions belong to the representative household, the average disutility function per family is $\upsilon_t \equiv \int_0^1 \upsilon_t(z)\, dz$.[7]

Even though members out of the labor market consume $C_{\mathfrak{c},t}$ units of each type-c consumption bundle, they make no monetary contribution to the household budget. However, being out of the labor market might be an optimal decision if being unemployed is a burden. Indeed, searching for a job is time consuming and annoying. This rationale justifies involuntary unemployment, and may be one of the causes for leaving the labor market.

A simple way to capture this phenomenon is to assume that the burden of being unemployed generates extra disutility $\upsilon_t^{\mathsf{u}} \mathsf{u}_t^e$ to the household, i.e. $\upsilon_t^{\mathsf{u}} \mathsf{u}_t^e \equiv \mathfrak{w}_{\mathfrak{m}} \bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}} \mathsf{u}_{\mathfrak{m},t}^e + \mathfrak{w}_{\mathfrak{s}} \bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}} \mathsf{u}_{\mathfrak{s},t}^e$, where $\bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}}$ and $\bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}}$ are fixed sector-specific homogeneous disutility variables faced by unemployed workers. In this case, members out of the labor market contribute for the household by avoiding extra disutilities. In the end of the day, a trade-off arises because leaving the labor market also reduces the number of job matches and, as a consequence, reduces the expected household income.

The representative household maximizes its welfare measure $\mathcal{U}_t = \max \ \left(\ell_t^{\mathfrak{p}} u_t - \upsilon_t - \upsilon_t^{\mathsf{u}} \mathsf{u}_t^e\right) + E_t \beta \mathcal{U}_{t+1}$, subject to the budget constraint and the equations related to the labor market (not shown for being irrelevant for now). Let λ_t denote the Lagrange multiplier on the nominal budget constraint. For simplification, I aggregate unemployment compensations with the unemployment disutilities into what I call net unemployment compensations $\varpi_{\mathfrak{m},t}^u$ and $\varpi_{\mathfrak{s},t}^u$, defined as $\varpi_{\mathfrak{m},t}^u \equiv \varpi_{\mathfrak{m},t}^c - \frac{\bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}}}{\lambda_t P_t}$ and $\varpi_{\mathfrak{s},t}^u \equiv \varpi_{\mathfrak{s},t}^c - \frac{\bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}}}{\lambda_t P_t}$.

The aggregate net unemployment compensation ϖ_t^u is defined as $\varpi_t^u \equiv \frac{1}{\mathsf{u}_t^e} \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varpi_{\mathfrak{c},t}^u \mathsf{u}_{\mathfrak{c},t}^e$.

Therefore, the representative household chooses C_t, A_{t+1}, and $\mathcal{B}_{t+1}$ to solve:

$$\begin{aligned} \mathcal{U}_t = \max \ \ell_t^{\mathfrak{p}} u_t - \upsilon_t + \lambda_t \Big[& A_t + \mathcal{B}_t I_{t-1} + P_t \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varpi_{\mathfrak{c},t}^u \mathsf{u}_{\mathfrak{c},t}^e - \Xi_t + P_t d_t \\ & + \textstyle\sum_{\mathfrak{c}} \int_{\mathfrak{c}} \mathsf{n}_t(z_{\mathfrak{c}})\, \mathcal{W}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}} - \ell_t^{\mathfrak{p}} P_t C_t - E_t Q_{t+1} A_{t+1} - \mathcal{B}_{t+1} \Big] + E_t \beta \mathcal{U}_{t+1} \end{aligned}$$

where d_t denotes real profits from all firms, Ξ_t denotes lump-sum taxes net of transfers from the government, B_t is the value of one-period non-contingent domestic bonds at the end of period t, $I_t \equiv 1 + i_t$ is the gross interest rate on the domestic bond, A_t is the aggregate state-contingent value of the portfolio of financial securities held at the beginning of period t, E_t is the time-t expectations operator, and Q_{t+1} is the stochastic discount factor from $t+1$ to t.

[7] Using a unions-based aggregate disutility function instead of a workers-based one allows me to derive closed form equations describing the dynamics of the aggregate disutility to work in Section 2.4, which is an important variable for understanding the amplified volatilities under trend inflation. The dynamics implied by the labor flows and by the Calvo price setting convolute in such a way that the derivation is not possible otherwise. The unions-based disutility also allows me to obtain the firms' supply equations with no need to guess the loglinearized function forms to deal with the issue on firms' specific labor, as done in Thomas (2008).

The first-order conditions are the non-arbitrage condition $E_t Q_{t+1} = 1/I_t$, and the Euler equations

$$1 = \beta E_t \left(\frac{u'_{t+1}}{u'_t} \frac{I_t}{\Pi_{t+1}} \right) \quad ; \; Q_t = \beta \frac{u'_t}{u'_{t-1}} \frac{1}{\Pi_t} \tag{27}$$

where $u'_t \equiv \mathrm{u}_{\mathrm{u},t} \left(C_t - \iota_{\mathrm{u}} C_{t-1} \right)^{-\sigma}$ is the marginal utility to consumption. in equilibrium, $\Pi_t \equiv 1 + \pi_t$ is the gross inflation rate, and $\lambda_t = u'_t / P_t$ is the Lagrange multiplier on the budget constraint.

In equilibrium, demand for financial securities matches their supply by individuals, so that the aggregate state-contingent value of the portfolio held at the beginning of period t is $A_t = 0$, $\forall t$.

2.2.3 Leaving the labor force

Before deriving the optimal rules for sectoral migration, I present some comments and definitions. Individuals take the predetermined variables n_t, u_t, v_t, p_t and q_t, and their sectoral peers, as given. In this context, $\theta^f_t \equiv \theta_{t+1}$, $\mathsf{p}^f_t \equiv \mathsf{p}_{t+1}$, $\mathsf{q}^f_t \equiv \mathsf{q}_{t+1}$, and their sectoral peers, are key in deriving the optimal masses out of the labor forces in this section and the wage and the aggregate job market curves in Section ??.[8]

The job-finding rate for being matched at firm $z_{\mathfrak{c}}$, at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$, is $\mathsf{p}_t(z_{\mathfrak{c}}) \equiv \mathsf{m}_t(z_{\mathfrak{c}}) / (\mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t})$. The rate satisfies $\mathsf{p}_{\mathfrak{c},t} = \int_{\mathfrak{c}} \mathsf{p}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$. Likewise, the firm's vacancy share in the sector is $\mathsf{s}_t(z_{\mathfrak{c}}) \equiv \mathsf{v}_t(z_{\mathfrak{c}}) / (\mathfrak{w}_{\mathfrak{c}} \mathsf{v}_{\mathfrak{c},t})$. Note that $\mathsf{s}_t(z_{\mathfrak{c}})$ also equals $\mathsf{p}_t(z_{\mathfrak{c}}) / \mathsf{p}_{\mathfrak{c},t}$, the probability that the worker is matched into firm $z_{\mathfrak{c}}$, conditioned on obtaining a new job in the sector. It implies that $\int_{\mathfrak{c}} \mathsf{s}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}} = 1$.

Finally, the mass of workers matched into firm $z_{\mathfrak{c}}$ at period $t+1$ can be computed as follows:

$$\mathsf{m}_{t+1}(z_{\mathfrak{c}}) = \mathsf{q}_{\mathfrak{c},t+1} \mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \frac{\mathsf{m}_{\mathfrak{c},t+1}}{\mathsf{v}_{\mathfrak{c},t+1}} \mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \frac{\mathsf{p}_{\mathfrak{c},t+1} \mathsf{u}_{\mathfrak{c},t+1}}{\mathsf{v}_{\mathfrak{c},t+1}} \mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \mathfrak{w}_{\mathfrak{c}} \mathsf{s}_{t+1}(z_{\mathfrak{c}})\, \mathsf{p}_{\mathfrak{c},t+1} \mathsf{u}_{\mathfrak{c},t+1}$$

For notation purposes, let $\upsilon'_t(z_{\mathfrak{c}}) \equiv \partial \upsilon_t(z_{\mathfrak{c}}) / \partial H_t(z_{\mathfrak{c}}) = (1+\nu)\, \upsilon_t(z_{\mathfrak{c}}) / H_t(z_{\mathfrak{c}})$.

Individuals may lack full information when deciding on whether leaving the labor market or reallocating to the other sector, considering myopic expectations on future flows $\tilde{E}_t \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}$ as given, which match the aggregate expectation in equilibrium $E_t \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}$. In order to capture this phenomenon, I assume that the household faces additional real adjustment costs $\frac{\varsigma_{\mathrm{m}\mathfrak{c}}}{2} \left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\tilde{E}_t \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}} - 1 \right)^2$ on changes of the masses of unemployed workers leaving the labor market. I also assume that myopic expectations clears in equilibrium, i.e. $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} / \tilde{E}_t \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1} = E_t \left(\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} / \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1} \right)$.

Let me now rewrite the representative family's problem, including extra restrictions from labor flows and using the notation of net unemployment compensations $\varpi^u_{\mathfrak{c},t}$. They do not bind previously derived first order conditions, and hence are not included for computing the optimal consumption allocation. The additional restrictions are the ones described by equations (8), (9), (10), (11), (12), (5).

Therefore, when deciding the optimal mass of unemployed workers to leave the labor market or reallocate to a different sector, the representative household chooses $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$, $\ell_{\mathfrak{c},t}$, $\ell^{\mathfrak{o}}_{\mathfrak{c},t}$, $\mathsf{u}_{\mathfrak{c},t+1}$ and $\mathsf{n}_{t+1}(z_{\mathfrak{c}})$ to maximize

[8] Note that end-of-period variables θ^e_t, p^e_t and q^e_t are not the same as lead variables θ^f_t, p^f_t and q^f_t.

the same expected discounted utility flow, conditioned on those extra restrictions. Let Q_t^{π}, defined below, denote the real stochastic discount factor. Recall also that $\lambda_t P_t = u'_t$, where u'_t is the marginal utility to consumption. Therefore, the first order conditions to pin down $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$ can be simplified to:

$$\mathfrak{w}_{\mathfrak{c}}\varpi^{\mathfrak{o}}_{\mathfrak{c},t} - \mathfrak{w}_{\mathfrak{c}}\varpi^{\ell}_{\mathfrak{c},t} - \varsigma_{\mathsf{mc}}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} - 1\right)\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} + \varsigma_{\mathsf{mc}}E_t Q^{\pi}_{t+1}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}} - 1\right)\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}\right)\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}} = 0 \tag{28}$$

$$\varpi^{\ell}_{\mathfrak{c},t} = \varpi^{u}_{\mathfrak{c},t} + (1-\rho_{\mathfrak{d}})\,\Lambda^{\mathsf{u}}_{\mathfrak{c},t} \tag{29}$$

$$\varpi^{\mathfrak{o}}_{\mathfrak{c},t} = (1-\rho_{\mathfrak{d}})\left(\delta^{\mathfrak{c}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\mathfrak{c},t} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\bar{\mathfrak{c}},t}\right) + (1-\rho_{\mathfrak{d}})\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)E_t Q^{\pi}_{t+1}\varpi^{\mathfrak{o}}_{\mathfrak{c},t+1} \tag{30}$$

$$\Lambda^{\mathsf{u}}_{\mathfrak{c},t} = a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}E_t\int_{\mathfrak{c}}\Lambda^{cn}_t(z_{\mathfrak{c}})\,\mathsf{s}_{t+1}(z_{\mathfrak{c}})\,dz_{\mathfrak{c}} + E_t Q^{\pi}_{t+1}\left(1-a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}\right)\varpi^{\ell}_{\mathfrak{c},t+1} \tag{31}$$

$$\begin{aligned}\Lambda^{cn}_t(z_{\mathfrak{c}}) \quad = \quad & E_t Q^{\pi}_{t+1}\left[-\frac{v'_{t+1}(z_{\mathfrak{c}})\,h_{t+1}(z_{\mathfrak{c}})}{u'_{t+1}} + \varpi_{t+1}(z_{\mathfrak{c}}) - \varpi^{u}_{\mathfrak{c},t+1}\right] \\ & + E_t Q^{\pi}_{t+1}\left[\varpi^{\ell}_{\mathfrak{c},t+1} + (1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t+1}\right)\left(\Lambda^{cn}_{t+1}(z_{\mathfrak{c}}) - \Lambda^{\mathsf{u}}_{\mathfrak{c},t+1}\right)\right]\end{aligned} \tag{32}$$

where

$$\varpi^{\mathfrak{o}}_{\mathfrak{c},t} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\tfrac{\lambda^{\mathfrak{o}}_{\mathfrak{c},t}}{u'_t} \quad \varpi^{\ell}_{\mathfrak{c},t} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\tfrac{\lambda^{\ell}_{\mathfrak{c},t}}{u'_t} \quad ; \ \Lambda^{\mathsf{u}}_{\mathfrak{c},t} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\tfrac{\lambda^{\mathsf{u}}_{\mathfrak{c},t}}{u'_t} \quad ; \ \Lambda^{cn}_t(z_{\mathfrak{c}}) \equiv \tfrac{\lambda^{cn}_t(z_{\mathfrak{c}})}{u'_t} \quad ; \ Q^{\pi}_t \equiv Q_t\Pi_t \tag{33}$$

2.3 Firms

Goods are produced in sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Each firm produces with technology

$$y_t(z_{\mathfrak{c}}) = \mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t H_t(z_{\mathfrak{c}})^{\varepsilon_{\mathfrak{c}}} \tag{34}$$

where $\varepsilon_{\mathfrak{c}} \in (0,1)$, $H_t(z_{\mathfrak{c}}) \equiv h_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})$ is the total hours worked, A_t is the aggregate technology shock, and $a_{\mathfrak{c},t}$ is the sector-c idiosyncratic technology shock.

The technology shocks A_t and $\mathsf{a}_{\mathfrak{c},t}$ are stationary exogenous processes, described by $\frac{\mathsf{A}_t}{\bar{\mathsf{A}}} = \epsilon_{\mathsf{A},t}\left(\frac{\mathsf{A}_{t-1}}{\bar{\mathsf{A}}}\right)^{\phi_{\mathsf{A}}}$ and $\frac{\mathsf{a}_{\mathfrak{c},t}}{\bar{\mathsf{a}}_{\mathfrak{c}}} = \epsilon^{\mathsf{a}}_{\mathfrak{c},t}\left(\frac{\mathsf{a}_{\mathfrak{c},t-1}}{\bar{\mathsf{a}}_{\mathfrak{c}}}\right)^{\phi^{\mathsf{a}}_{\mathfrak{c}}}$, where $\epsilon_{\mathsf{A},t}$ and $\epsilon^{\mathsf{a}}_{\mathfrak{c},t}$ are the aggregate and sector-c idiosyncratic technology innovations, $\phi_{\mathsf{A}} \in (0,1)$ and $\phi^{\mathsf{a}}_{\mathfrak{c}} \in (0,1)$.

Let $P_{\mathfrak{c},t}\mathcal{Y}_{\mathfrak{c},t} \equiv \int_{\mathfrak{c}} p_t(z_{\mathfrak{c}})\,y_t(z_{\mathfrak{c}})\,dz_{\mathfrak{c}}$ denote the sector-$\mathfrak{c}$ gross output, i.e. the aggregate revenue from sales, and $\mathcal{Y}_t$ denote the economy gross output $\mathcal{Y}_t \equiv \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t}\mathcal{Y}_{\mathfrak{c},t}$.

Firms' market clearing conditions are $y_t(z_{\mathfrak{c}}) = \boldsymbol{c}_t(z_{\mathfrak{c}})$, $\mathcal{Y}_{\mathfrak{c},t} = \mathfrak{C}_{\mathfrak{c},t}$ and $\mathcal{Y}_t = \mathfrak{C}_t$, where $\mathfrak{C}_{\mathfrak{c},t}$ and $\mathfrak{C}_t$ are again the sectoral and the economy wide consumption bundles, as defined in Section 2.2.1. Using the market clearing conditions and the demand functions, I obtain the demand functions:

$$y_t(z_{\mathfrak{c}}) = \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi} \quad ; \ \mathcal{Y}_{\mathfrak{c},t} = \mathfrak{w}_{\mathfrak{c}}\mathfrak{C}_t\left(\wp_{\mathfrak{c},t}\right)^{-\phi} \tag{35}$$

As mentiond in Section 2.2.1, $\mathsf{c}_{\mathfrak{m},t}^{\mathsf{vc}}$ and $\mathsf{c}_{\mathfrak{s},t}^{\mathsf{vc}}$ represent firms' intermediate consumption and must be netted out when computing sector-c and the economy-wide GDP's, defined as $Y_{\mathfrak{c},t} \equiv C_{\mathfrak{c},t}$ and $Y_t \equiv C_t$.

2.3.1 Wage bargaining

Recall that each firm $z_{\mathfrak{c}}$ at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$ uses labor in both the intensive $h_t(z_{\mathfrak{c}})$ and extensive $\mathsf{n}_t(z_{\mathfrak{c}})$ margins according to the technology $y_t(z_{\mathfrak{c}}) = \mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t H_t(z_{\mathfrak{c}})^{\varepsilon_{\mathfrak{c}}}$, where $H_t(z_{\mathfrak{c}}) = h_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})$.

The total real salary per period $\varpi_t(z_{\mathfrak{c}}) = w_t(z_{\mathfrak{c}})\,h_t(z_{\mathfrak{c}})$ and hours per worker $h_t(z_{\mathfrak{c}})$ are decided by Nash bargaining and maximize $b_{\mathfrak{c},t}\log(\mathfrak{U}_t(z_{\mathfrak{c}})) + (1-b_{\mathfrak{c},t})\log(\mathfrak{J}_t(z_{\mathfrak{c}}))$, where $\mathfrak{U}_t(z_{\mathfrak{c}})$ and $\mathfrak{J}_t(z_{\mathfrak{c}})$ are the worker's and firm's real match surpluses when the marginal worker is matched into firm $z_{\mathfrak{c}}$. As in Ravenna and Walsh (2011), I assume that the workers' bargaining power $b_{\mathfrak{c},t}$ is time-varying and evolves according to a stationary process about its steady state level $\bar{b}_{\mathfrak{c}}$, $\frac{b_{\mathfrak{c},t}}{\bar{b}_{\mathfrak{c}}} = \epsilon_{\mathfrak{c},t}^b\left(\frac{b_{\mathfrak{c},t-1}}{\bar{b}_{\mathfrak{c}}}\right)^{\phi_{\mathfrak{c}}^b}$, where $\epsilon_{\mathfrak{c},t}^b$ is the sector-c specific shock on the bargaining power and $\phi_{\mathfrak{c}}^b \in (0,1)$.

I derive the aggregate wage curve and the aggregate job creation curve, shown below. My analysis departs from Thomas (2011) and Alves (2012) by assuming that hours must be set to maximize the total surplus, as will be optimal both from the firm's as the union's perspectives, and not assuming that firms internalize the existence of a wage schedule, as a function of only hours, prior to optimization.

For notation purposes, let $\upsilon_t'(z_{\mathfrak{c}}) \equiv \partial\upsilon_t(z_{\mathfrak{c}})/\partial H_t(z_{\mathfrak{c}})$ denote the marginal disutility to work, $\varpi_t'(z_{\mathfrak{c}}) \equiv \partial\varpi(h_t(z_{\mathfrak{c}}))/\partial h_t(z_{\mathfrak{c}})$ denote the marginal real salary, $w_t'(z_{\mathfrak{c}}) \equiv \partial w(h_t(z_{\mathfrak{c}}))/\partial h_t(z_{\mathfrak{c}})$ denote the marginal real wage. Recall also that $\lambda_t P_t = u_t'$, where u_t' is the marginal utility to consumption, and $Q_t^{\pi} \equiv Q_t\Pi_t$ is the real stochastic discount factor.

Bargaining takes place taking the extensive margin $\mathsf{n}_t(z_{\mathfrak{c}})$ as given, as soon as new hired workers arrive, in the beginning of period t, slightly after prices $p_t(z_{\mathfrak{c}})$ are set. Therefore, due to the demand function, total current revenue $\bar{\mathcal{R}}_t(z_{\mathfrak{c}})$ is also given. Therefore, considering the law of motion of its employment stock, the firm chooses $\mathsf{v}_t^e(z_{\mathfrak{c}})$ and $\mathsf{n}_{t+1}(z_{\mathfrak{c}})$ to maximize its expected present discounted sum of nominal profits $\mathcal{J}_t(z_{\mathfrak{c}})$:

$$\begin{aligned}
\mathcal{J}_t(z_{\mathfrak{c}}) = \max &\left[\bar{\mathcal{R}}_t(z_{\mathfrak{c}}) - P_t w_t(z_{\mathfrak{c}})\,H_t(z_{\mathfrak{c}}) - P_t\varsigma_{\mathsf{vc}}\mathsf{v}_t^e(z_{\mathfrak{c}})\right] + E_t Q_{t+1}\mathcal{J}_{t+1}(z_{\mathfrak{c}}) \\
&+ P_t\lambda_t^n(z_{\mathfrak{c}})\,E_t\left[(1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t}\right)\mathsf{n}_t(z_{\mathfrak{c}}) + \mathsf{q}_{\mathfrak{c},t}^f\mathsf{v}_t^e(z_{\mathfrak{c}}) - \mathsf{n}_{t+1}(z_{\mathfrak{c}})\right] \\
&+ \lambda_t^r(z_{\mathfrak{c}})\left[\mathcal{R}_t(z_{\mathfrak{c}}) - \bar{\mathcal{R}}_t(z_{\mathfrak{c}})\right]
\end{aligned}$$

where $\mathcal{R}_t(z_{\mathfrak{c}}) \equiv p_t(z_{\mathfrak{c}})\,y_t(z_{\mathfrak{c}})$ is the revenue function.

The first order conditions are $\lambda_{\mathfrak{c},t}^n \equiv \lambda_t^n(z_{\mathfrak{c}}) = \frac{\varsigma_{\mathsf{vc}}}{\mathsf{q}_{\mathfrak{c},t}^f}$ (for $\mathsf{v}_t^e(z_{\mathfrak{c}})$) and $\lambda_t^n(z_{\mathfrak{c}}) = E_t Q_{t+1}^{\pi}\mathfrak{J}_{t+1}(z_{\mathfrak{c}})$ (for $\mathsf{n}_{t+1}(z_{\mathfrak{c}})$), where $\mathfrak{J}_t(z_{\mathfrak{c}}) \equiv \frac{1}{P_t}\frac{\partial\mathcal{J}_t(z_{\mathfrak{c}})}{\partial\mathsf{n}_t(z_{\mathfrak{c}})}$ is the real value of the marginal worker to the firm, i.e. the firm's real match surplus, which is computed by means of the Envelope Theorem. For that, I use the production (34), demand (35) and revenue functions. Independently of the firms type, the real value of the marginal worker to the firm can be written as follows:

$$\mathfrak{J}_t(z_\mathfrak{c}) = \frac{1}{\mu}\frac{p_t(z_\mathfrak{c})}{P_t}\frac{\varepsilon_\mathfrak{c} y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})}\lambda_t^r(z_\mathfrak{c}) - \varpi_t(z_\mathfrak{c}) + \left(1-\rho_{\mathfrak{dc},t}\right)\frac{\varsigma_{\mathsf{vc}}}{\mathsf{q}_{\mathfrak{c},t}^f} \tag{36}$$

where $\rho_{\mathfrak{dc},t} \equiv 1-(1-\rho_\mathfrak{d})\left(1-\rho_{\mathfrak{c},t}\right)$.

Let $\mathfrak{U}_t(z_\mathfrak{c}) \equiv \frac{1}{\lambda_t P_t}\frac{\partial \mathcal{U}_t}{\partial \mathsf{n}_t(z_\mathfrak{c})}$ denote the real match surplus enjoyed by the marginal worker matched into firm $z_\mathfrak{c}$, in monetary units. The solution to the Nash bargaining is $\frac{\mathfrak{U}_t(z_\mathfrak{c})}{\mathfrak{J}_t(z_\mathfrak{c})} = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}$,[9] which implies that the total surplus $\mathfrak{T}_t(z_\mathfrak{c}) \equiv \mathfrak{U}_t(z_\mathfrak{c})+\mathfrak{J}_t(z_\mathfrak{c})$ is proportional to $\mathfrak{U}_t(z_\mathfrak{c})$ and $\mathfrak{J}_t(z_\mathfrak{c})$. This result implies that the household's and firms' Lagrange multipliers $\lambda_t^{cn}(z_\mathfrak{c}) = u_t'\Lambda_t^{cn}(z_\mathfrak{c})$ and $\lambda_t^n(z_\mathfrak{c})$ on the laws of motion of employment must satisfy:

$$\Lambda_t^{cn}(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\lambda_t^n(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\frac{\varsigma_{\mathsf{vc}}}{\mathsf{q}_{\mathfrak{c},t}^f} \tag{37}$$

Plugging (37) into (31), I obtain:

$$\Lambda_{\mathfrak{c},t}^{\mathsf{u}} = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}a_\mathfrak{c}\varsigma_{\mathsf{vc}}\theta_{\mathfrak{c},t}^f + (1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})E_t Q_{t+1}^\pi \varpi_{\mathfrak{c},t+1}^\ell \tag{38}$$

Since $\mathfrak{U}_t(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\mathfrak{J}_t(z_\mathfrak{c})$, I obtain the equations describing the evolution dynamics of the firm's salary as a function of $\lambda_t^r(z_\mathfrak{c})$:

$$\begin{aligned}\varpi_t(z_\mathfrak{c}) \;=\;& (1-b_{\mathfrak{c},t})\varpi_{\mathfrak{c},t}^u + (1-b_{\mathfrak{c},t})(1+\nu)\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u_t'} + b_{\mathfrak{c},t}\frac{1}{\mu}\frac{p_t(z_\mathfrak{c})}{P_t}\frac{\varepsilon_\mathfrak{c} y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})}\lambda_t^r(z_\mathfrak{c})\\ &+b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{dc},t}\right)a_\mathfrak{c}\varsigma_{\mathsf{vc}}\theta_{\mathfrak{c},t}^f + (1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{dc},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})E_t Q_{t+1}^\pi\varpi_{\mathfrak{c},t+1}^\ell - \varpi_{\mathfrak{c},t}^\ell\right]\end{aligned}$$

Since $\mathfrak{J}_t(z_\mathfrak{c})$ and $\mathfrak{U}_t(z_\mathfrak{c})$ are proportional to $\mathfrak{T}_t(z_\mathfrak{c})$, both the firm and union agree to choose hours $h_t(z_\mathfrak{c})$ to have the total surplus maximized. Therefore, I pin down optimal $\lambda_t^r(z_\mathfrak{c})$ as follows:

$$\frac{1}{\mu}\frac{p_t(z_\mathfrak{c})}{P_t}\frac{\varepsilon_\mathfrak{c} y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})}\lambda_t^r(z_\mathfrak{c}) = \frac{(1+\nu)^2}{\varepsilon_\mathfrak{c}}\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u_t'}$$

Plugging this result in the salary equation, I obtain the firm's salary curve:

$$\begin{aligned}\varpi_t(z_\mathfrak{c}) \;=\;& (1-b_{\mathfrak{c},t})\varpi_{\mathfrak{c},t}^u + (1-b_{\mathfrak{c},t})\mathfrak{z}_{1\mathfrak{c},t}\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u_t'} + b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{dc},t}\right)a_\mathfrak{c}\varsigma_{\mathsf{vc}}\theta_{\mathfrak{c},t}^f\\ &+(1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{dc},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})E_t Q_{t+1}^\pi\varpi_{\mathfrak{c},t+1}^\ell - \varpi_{\mathfrak{c},t}^\ell\right]\end{aligned}$$

where $\mathfrak{z}_{1\mathfrak{c},t} \equiv (1+\nu)\left[1+\tilde{b}_{\mathfrak{c},t}(1+\omega_\mathfrak{c})\right]$, $\omega_\mathfrak{c} \equiv \frac{1+\nu}{\varepsilon_\mathfrak{c}}-1$ and $\tilde{b}_{\mathfrak{c},t} \equiv \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}$.

It implies that the marginal salary is $\varpi_t'(z_\mathfrak{c}) = \mathfrak{z}_{2\mathfrak{c},t}\frac{\upsilon_t'(z_\mathfrak{c})}{u_t'}$, where $\mathfrak{z}_{2\mathfrak{c},t} \equiv (1-b_{\mathfrak{c},t})\mathfrak{z}_{1\mathfrak{c},t}$. The firm's job creation curve is then obtained by plugging those results into the firm's first order condition: $\frac{\varsigma_{\mathsf{vc}}}{\mathsf{q}_{\mathfrak{c},t}^f} =$

[9] The general solution of the Nash bargaining is $\frac{\mathfrak{U}_t(z_\mathfrak{c})}{\mathfrak{J}_t(z_\mathfrak{c})} = -\frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\frac{\partial\mathfrak{U}_t(z_\mathfrak{c})/\partial\varpi_t(z_\mathfrak{c})}{\partial\mathfrak{J}_t(z_\mathfrak{c})/\partial\varpi_t(z_\mathfrak{c})}$.

$E_t Q^{\pi}_{t+1}\left[\mathfrak{z}_{3\mathfrak{c}}\frac{\upsilon_{t+1}(z_\mathfrak{c})/\mathsf{n}_{t+1}(z_\mathfrak{c})}{u'_{t+1}} - \varpi_{t+1}(z_\mathfrak{c}) + \left(1-\rho_{\mathfrak{d}\mathfrak{c},t+1}\right)\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t+1}}\right]$, where $\mathfrak{z}_{3\mathfrak{c}} \equiv (1+\nu)(1+\omega_\mathfrak{c})$.

Interestingly, note that (32) becomes a redundant result once we consider the system described by the firm's salary and job creation curves. Let $\upsilon_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c} \upsilon_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ and $\varpi_{\mathfrak{c},t} \equiv \frac{1}{\mathsf{n}_{\mathfrak{c},t}}\frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c} \varpi_t(z_\mathfrak{c})\,n_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ denote the aggregate disutility and the the aggregate salary at sector $\mathfrak{c} \in \mathcal{F}_\mathfrak{c}$. Integrating over all firms in this sector, I obtain the following sectoral aggregate salary curve and aggregate job creation curve:

$$\begin{aligned}\varpi_{\mathfrak{c},t} = {} & (1-b_{\mathfrak{c},t})\,\varpi^u_{\mathfrak{c},t} + (1-b_{\mathfrak{c},t})\,\mathfrak{z}_{1\mathfrak{c},t}\frac{\upsilon_{\mathfrak{c},t}/\mathsf{n}_{\mathfrak{c},t}}{u'_t} + b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)a_\mathfrak{c}\varsigma_{\mathsf{v}\mathfrak{c}}\theta^f_{\mathfrak{c},t}\\ + {} & (1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})\,E_tQ^\pi_{t+1}\varpi^\ell_{\mathfrak{c},t+1} - \varpi^\ell_{\mathfrak{c},t}\right]\end{aligned} \tag{39}$$

$$\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}} = E_tQ^\pi_{t+1}\left[\mathfrak{z}_{3\mathfrak{c}}\frac{\upsilon_{\mathfrak{c},t+1}/\mathsf{n}_{\mathfrak{c},t+1}}{u'_{t+1}} - \varpi_{\mathfrak{c},t+1} + \left(1-\rho_{\mathfrak{d}\mathfrak{c},t+1}\right)\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t+1}}\right] \tag{40}$$

In this context, the economy wide aggregate salary ϖ_t is defined as follows:

$$\varpi_t = \frac{1}{\mathsf{n}_t}\sum_\mathfrak{c}\mathfrak{w}_\mathfrak{c}\varpi_{\mathfrak{c},t}\mathsf{n}_{\mathfrak{c},t} \tag{41}$$

2.3.2 Production firms - Marginal costs

The firm internalizes the fact that the marginal real salary is a function of its own production level $y_t(z_\mathfrak{c})$. Therefore, the real cost of a domestic firm $z_\mathfrak{c}$ at sector $\mathfrak{c} \in \mathcal{F}_\mathfrak{c} \equiv \{\mathfrak{m},\mathfrak{s}\}$ is

$$\mathfrak{Cost}_t(z_\mathfrak{c}) = \mathfrak{s}_{\mathfrak{c},t}\varpi_t(z_\mathfrak{c})\,\mathsf{n}_t(z_\mathfrak{c}) + \varsigma_{\mathsf{v}\mathfrak{c}}\mathsf{v}^e_t(z_\mathfrak{c})$$

where $\mathfrak{s}_{\mathfrak{c},t}$ is an additional sector-$\mathfrak{c}$ specific cost shock over payroll, not properly taken into account during the bargaining process, which evolves according to a stationary process about its steady state level $\bar{\mathfrak{s}}_\mathfrak{c}$, i.e. $\frac{\mathfrak{s}_{\mathfrak{c},t}}{\bar{\mathfrak{s}}_\mathfrak{c}} = \epsilon^\mathfrak{s}_{\mathfrak{c},t}\left(\frac{\mathfrak{s}_{\mathfrak{c},t-1}}{\bar{\mathfrak{s}}_\mathfrak{c}}\right)^{\phi^\mathfrak{s}_\mathfrak{c}}$, where $\epsilon^\mathfrak{s}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ specific innovation on the cost shock and $\phi^\mathfrak{s}_\mathfrak{c} \in (0,1)$.

The firm is free to adjust the intensive margin $h_t(z_\mathfrak{c})$. The extensive margin $\mathsf{n}_t(z_\mathfrak{c})$, however, depends only on previous decisions and hence is fixed during the period. Therefore, the real marginal cost is computed as $mc_t(z_\mathfrak{c}) = \varrho_\mathfrak{c}\epsilon^{\mathfrak{mc}}_{\mathfrak{c},t}(C_t - \iota_\mathfrak{u}C_{t-1})^\sigma(y_t(z_\mathfrak{c}))^{\omega_\mathfrak{c}}$, here $\varrho_\mathfrak{c} \equiv \frac{\chi}{\varepsilon_\mathfrak{c}}$ and $\epsilon^{\mathfrak{mc}}_{\mathfrak{c},t} \equiv \mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}(\mathfrak{u}_{\mathfrak{u},t})^{-1}(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-(1+\omega_\mathfrak{c})}$.

2.3.3 Price setting

With probability $(1-\alpha_\mathfrak{c})$, firm $z_\mathfrak{c}$ optimally readjusts its selling price to $p_t(z_\mathfrak{c}) = \bar{p}_{\mathfrak{c},t}$. With probability $\alpha_\mathfrak{c}$, its price is adjusted to $p_t(z_\mathfrak{c}) = p_{t-1}(z_\mathfrak{c})\,\Pi^{ind}_{\mathfrak{c},t}$, where $\Pi^{ind}_{\mathfrak{c},t} = (\Pi_{\mathfrak{c},t-1})^{\iota_\mathfrak{c}}$ and $\iota_\mathfrak{c} \in (0,1)$ is the indexation degree. When optimally readjusting, firm $z_\mathfrak{c}$ sets its price to maximize its expected present discounted sum of profits, subject to the demand and marginal functions. The first order condition implies $\left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{(1+\phi\omega_\mathfrak{c})} = \frac{\mathcal{N}_{\mathfrak{c},t}}{\mathcal{D}_{\mathfrak{c},t}}$, where

$\mu = \frac{\phi}{(\phi-1)}$, $\mathcal{G}_{\mathfrak{c},t} \equiv \frac{\mathcal{Y}_{\mathfrak{c},t}}{\mathcal{Y}_{\mathfrak{c},t-1}}$.and

$$\begin{array}{ll} \mathcal{N}_{\mathfrak{c},t} = \mu \varrho_{\mathfrak{c}} \epsilon_{\mathfrak{c},t}^{\mathfrak{mc}} \left(\wp_{\mathfrak{c},t}\right)^{-1} \left(C_t - \iota_{\mathfrak{u}} C_{t-1}\right)^{\sigma} \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{\omega_{\mathfrak{c}}} + \alpha_{\mathfrak{c}} E_t \mathfrak{n}_{\mathfrak{c},t+1} & ; \ \mathcal{D}_{\mathfrak{c},t} = 1 + \alpha_{\mathfrak{c}} E_t \mathfrak{d}_{\mathfrak{c},t+1} \\ \mathfrak{n}_{\mathfrak{c},t} = Q_t \mathcal{G}_{\mathfrak{c},t} \Pi_{\mathfrak{c},t} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi_{\mathfrak{c},t}^{ind}}\right)^{\phi(1+\omega_{\mathfrak{c}})} \mathcal{N}_{\mathfrak{c},t} & ; \ \mathfrak{d}_{\mathfrak{c},t} = Q_t \mathcal{G}_{\mathfrak{c},t} \Pi_{\mathfrak{c},t} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi_{\mathfrak{c},t}^{ind}}\right)^{(\phi-1)} \mathcal{D}_{\mathfrak{c},t} \end{array}$$

The Calvo price setting structure implies $1 = (1-\alpha_{\mathfrak{c}}) \left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{-(\phi-1)} + \alpha_{\mathfrak{c}} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi_{\mathfrak{c},t}^{ind}}\right)^{(\phi-1)}$.

2.4 Relative prices, aggregates and productivity

Modelling aggregate disutility functions $\upsilon_t \equiv \int \upsilon_t(z)\, dz$ and $\upsilon_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \upsilon_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$, and aggregate hours worked $H_t \equiv \int H_t(z)\, dz$ and $H_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} H_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ requires more elaboration. Those variables can be rewritten as follows:

$$\begin{array}{ll} \upsilon_{\mathfrak{c},t} \equiv \frac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t} \mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{c}})} \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\omega_{\mathfrak{c}})} \left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})} & ; \ \upsilon_t = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \upsilon_{\mathfrak{c},t} \\ H_{\mathfrak{c},t} \equiv \left(\mathsf{a}_{\mathfrak{c},t} \mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{s}})} \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} & ; \ H_t = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} H_{\mathfrak{c},t} \end{array}$$

where $\tilde{\omega}_{\mathfrak{c}} \equiv \frac{1}{\varepsilon_{\mathfrak{c}}} - 1$, while $\mathcal{P}_{\upsilon\mathfrak{c},t}$ and $\mathcal{P}_{H\mathfrak{c},t}$ denote aggregate relative prices:

$$\left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\frac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\omega_{\mathfrak{c}})} dz_{\mathfrak{c}} \quad ; \ \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\frac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} dz_{\mathfrak{c}}$$

which evolve according to the following dynamics:

$$\begin{array}{rcl} \left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})} & = & (1-\alpha_{\mathfrak{c}}) \left(\bar{p}_{\mathfrak{c},t}/P_{\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})} + \alpha_{\mathfrak{c}} \left(\Pi_{\mathfrak{c},t}/\Pi_{\mathfrak{c},t}^{ind}\right)^{\phi(1+\omega_{\mathfrak{c}})} \left(\mathcal{P}_{\upsilon\mathfrak{c},t-1}\right)^{-\phi(1+\omega_{\mathfrak{c}})} \\ \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} & = & (1-\alpha_{\mathfrak{c}}) \left(\bar{p}_{\mathfrak{c},t}/P_{\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} + \alpha_{\mathfrak{c}} \left(\Pi_{\mathfrak{c},t}/\Pi_{\mathfrak{c},t}^{ind}\right)^{\phi(1+\tilde{\omega}_{\mathfrak{c}})} \left(\mathcal{P}_{H\mathfrak{c},t-1}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} \end{array}$$

Important indicators are the aggregate hours per worker h_t and $h_{\mathfrak{c},t}$, aggregate wages w_t and $w_{\mathfrak{c},t}$, and output per total hours ratios $\mathcal{A}_t$ and $\mathcal{A}_{\mathfrak{c},t}$. Those variables are defined as follows:

$$h_t \equiv \frac{H_t}{\mathsf{n}_t} \quad ; \ h_{\mathfrak{c},t} \equiv \frac{H_{\mathfrak{c},t}}{\mathsf{n}_{\mathfrak{c},t}} \quad ; \ w_t \equiv \frac{\varpi_t}{h_t} \quad ; \ w_{\mathfrak{c},t} \equiv \frac{\varpi_{\mathfrak{c},t}}{h_{\mathfrak{c},t}} \quad ; \ \mathcal{A}_t \equiv \frac{Y_t}{H_t} \quad ; \ \mathcal{A}_{\mathfrak{c},t} \equiv \frac{Y_{\mathfrak{c},t}}{H_{\mathfrak{c},t}}$$

2.5 Monetary policy

Since monetary policy has an important role in the dynamics of the model, I present its log-linearized structure here. The monetary authority is assigned a inflation target $\bar{\pi} \geq 0$ to pursuit and implements monetary policy according to the Taylor rule $\left(\frac{I_t}{\bar{I}}\right) = \mathfrak{u}_{\mathfrak{i},t} \left(\frac{I_{t-1}}{\bar{I}}\right)^{\varphi_i} \left[\left(E_t \frac{\bar{\Pi}_{t+1}}{\bar{\Pi}}\right)^{\varphi_\pi} \left(\frac{Y_{t-1}}{\bar{Y}}\right)^{\varphi_y}\right]^{1-\varphi_i}$, in which the response parameters φ_i, φ_π and φ_y are consistent with stability and determinacy in equilibria with rational expectations, and $\mathfrak{u}_{\mathfrak{i},t}$ is the monetary policy shock, which evolves according to $\frac{\mathfrak{u}_{\mathfrak{i},t}}{\bar{\mathfrak{u}}_{\mathfrak{i}}} = \epsilon_{\mathfrak{i},t} \left(\frac{\mathfrak{u}_{\mathfrak{i},t-1}}{\bar{\mathfrak{u}}_{\mathfrak{i}}}\right)^{\phi_{\mathfrak{i}}}$, where $\phi_{\mathfrak{i}} \in (0,1)$ and $\epsilon_{\mathfrak{i},t}$ is the monetary policy shock.

3 Inference

I estimate the log-linearized version of this model with a Bayesian technique and a Metropolis-Hastings (after Metropolis et al. (1953) and Hastings (1970)) MCMC algorithm, for which convergence occurred after about 1,250,000 draws from the sampler, after what I keep the next 1,250,000 draws for inference and Bayesian impulse response exercises. For inference, I considered 13 observed quarterly variables, from 2003:Q1 to 2014:Q4: manufacturing (detrended) GDP, services (detrended) GDP, tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the manufacturing sector), non-tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the services sector), working-age population, participation rate, employed workers at the manufacturing sector, employed workers at the services sector, hours per worker at the manufacturing sector, aggregate hours per worker, separation rate at the manufacturing sector, total mass of hired workers (adjusted for formality), and nominal interest rate.

Hours per worker at the manufacturing sector and the separation rate at the manufacturing sector are from the IBGE's Monthly Industrial Survey (PIMES). The total mass of hired workers are from the Brazilian General List of Employed and Unemployed (CAGED) workers, released by the Brazilian Ministry of Labor. Since it only refers to workers in formal sector, I adjust this measure using the formality rate from the IBGE's Employment Monthly Survey (PME). The nominal interest rate is the Brazilian Central Bank rate (Selic). All remaining variables are released by the Brazilian Institute of Geography and Statistics (IBGE). In particular, the labor market variables are obtained from the IBGE's Employment Monthly Survey (PME). As for the Brazilian CPI measures, I extract them from the Broad Consumer Price Index (IPCA).

I considered 13 shocks: 2 sector-c specific shocks on the separation rate $\left(\epsilon^{\rho}_{\mathfrak{m},t} \text{ and } \epsilon^{\rho}_{\mathfrak{s},t}\right)$, the shock to the mass of individuals coming to working-age $(\epsilon_{\ell,t})$, 2 sector-c specific shocks on the efficiency of the matching technology $\left(\epsilon^{\eta}_{\mathfrak{m},t} \text{ and } \epsilon^{\eta}_{\mathfrak{s},t}\right)$, the preference innovation $(\epsilon_{\mathfrak{u},t})$, 2 sector-$c$ specific innovations on the idiosyncratic technology shocks $\left(\epsilon^{\mathfrak{a}}_{\mathfrak{m},t} \text{ and } \epsilon^{\mathfrak{a}}_{\mathfrak{s},t}\right)$, 2 sector-$c$ specific shocks on the bargaining power $\left(\epsilon^{b}_{\mathfrak{m},t} \text{ and } \epsilon^{b}_{\mathfrak{s},t}\right)$, 2 sector-$c$ specific specific innovations on the marginal cost shocks $\left(\epsilon^{\mathfrak{s}}_{\mathfrak{m},t} \text{ and } \epsilon^{\mathfrak{s}}_{\mathfrak{s},t}\right)$, and the monetary policy shock $(\epsilon_{\mathfrak{i},t})$.

I calibrate a few of the parameters. As for the death rate, I have detrended the working-age population by considering two log-linear constant growth rates (before and after 2009). The resulting residual implied an average death rate of $\rho_{\mathfrak{d}} = 0.495$. The mass of manufacturing firms were calibrated using the average elasticity of manufacturing prices and GDP's on the aggregate deflator and real level of GDP, from 1996 on. As a consequence, I obtained $\mathfrak{w}_{\mathfrak{m}} = 0.460$ (note that the weight of tradable goods on the IPCA inflation rate of market prices during the same period is 0.446). I set the elasticity of substitution at $\phi = 7$, which implies a price markup of $\mu = 1.17$.[10] As for the level of trend inflation, I considered the long-run Brazilian inflation target of $\bar{\pi} = 4.5$, and assumed full indexation between past inflation and inflation target. All steady state levels of exogenous shocks were set at 1, i.e. I have calibrated $\bar{\mathfrak{u}}_{\mathfrak{c}} = \bar{\mathfrak{s}}_{\mathfrak{m}} = \bar{\mathfrak{s}}_{\mathfrak{s}} = \bar{\mathsf{A}} = \bar{\mathfrak{a}}_{\mathfrak{m}} = \bar{\mathfrak{a}}_{\mathfrak{s}} = 1$. I normalize and set the steady state level of hours per worker in the services sector at $\bar{h}_{\mathfrak{s}} = 1$. The steady state level

[10] This value is consistent with the range used in the literature. For instance, Ravenna and Walsh (2008) assumes $\mu = 1.20$, while Thomas (2011) uses $\mu = 1.15$.

of the participation rate is set at is sample average: $\bar{\mathfrak{r}} = 0.567$. The steady state level of the employment ratio of the manufacturing sector is set at is sample average: $\widetilde{\mathfrak{n}}^e_{\mathfrak{m}} \equiv \frac{\mathfrak{w}_{\mathfrak{m}} \bar{\mathfrak{n}}_{\mathfrak{m}}}{\bar{\mathfrak{n}}} = 0.249$. I also assume that the monetary policy shock $\mathfrak{u}_{\mathfrak{i},t}$ is a white noise, i.e. I impose $\phi_{\mathfrak{i}} = 0$. Finally, the subjective discount parameter was set at $\beta = 0.982$ in order to match the average ex-post real interest rate.

My standard approach is considering flat marginal prior distributions for all 38 estimated deep parameters and 13 standard deviations, i.e. all priors are set to be uniform distributions on very large support sets, so that inference is not biased at all by ill-designed prior distributions.

Tables 1 and 2 show the posterior estimation of the deep parameters, some key steady state levels such as $\bar{\mathfrak{p}}^{\mathfrak{u}e}_{\mathfrak{m}} \equiv \frac{\mathfrak{w}_{\mathfrak{m}} \bar{\mathfrak{u}}^e_{\mathfrak{m}}}{\bar{\mathfrak{u}}^e}$, and shocks standard deviations for the heterogeneous model. In order to ensure that $0 \leq \delta^{\mathfrak{c}}_{\mathfrak{c}} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} < 1$, I use the following normalized transformation $\bar{\delta}^{\bar{\mathfrak{c}}}_{\mathfrak{c}} \equiv \frac{\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}}{(1-\delta^{\mathfrak{c}}_{\mathfrak{c}})}$, which is bounded between 0 and 1. Figures 3 to 8 show the marginal posterior distributions along with each marginal prior distribution, for the heterogeneous model. Note that every parameter is well identified with sufficiently narrow intervals.

Among the estimated parameters for the labor market, the relevant central estimates suggest that: (i) workers from the manufacturing sector who are out of the labor market take longer to return ($\frac{1}{\delta^{\mathfrak{m}}_{\mathfrak{m}}} \approx \frac{1}{0.53} = 1.9$ quarters) than workers from the service sector ($\frac{1}{\delta^{\mathfrak{s}}_{\mathfrak{s}}} \approx \frac{1}{0.88} = 1.1$ quarters) ; (ii) workers from the manufacturing sector reallocate much faster to the service sector ($\frac{1}{\bar{\delta}^{\mathfrak{s}}_{\mathfrak{m}}(1-\delta^{\mathfrak{m}}_{\mathfrak{m}})} \approx \frac{1}{0.91(1-0.53)} = 2.3$ quarters) than workers from the services sector ($\frac{1}{\bar{\delta}^{\mathfrak{m}}_{\mathfrak{s}}(1-\delta^{\mathfrak{s}}_{\mathfrak{s}})} \approx \frac{1}{0.08(1-0.88)} = 26$ years) - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare; (iii) although unemployed workers from the service sector find it slightly easier to get a job than workers from the manufacturing sector $(a_{\mathfrak{s}} \approx 0.95 > a_{\mathfrak{m}} \approx 0.96)$, the workers' bargaining power in the manufacturing sector is much larger than the bargaining power in the service sector $\left(\bar{b}_{\mathfrak{m}} \approx 0.95 > \bar{b}_{\mathfrak{s}} \approx 0.55\right)$. As a result, the average salary in the service sector are much more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The results also suggest that salary bargaining is much more efficient in the manufacturing sector, as the marginal posterior distribution of $\bar{b}_{\mathfrak{m}}$ almost matches that of $a_{\mathfrak{m}}$.[11] Note also that the estimated values for those relevant labor market parameters $\left(a_{\mathfrak{c}} \text{ and } \bar{b}_{\mathfrak{c}}\right)$ are larger much larger in Brazil than what is found in developed countries.[12] I also find that the labor market is much tighter in the services sector than in the manufacturing sector $\left(\bar{\theta}^e_{\mathfrak{s}} \approx 1.78 > \bar{\theta}^e_{\mathfrak{m}} \approx 0.85\right)$, which means that firms in the services sector have much more vacancy openings, relative to unemployed workers, than those from the manufacturing sector. Since $a_{\mathfrak{s}}$ is only slightly larger than $a_{\mathfrak{m}}$, it is the labor market tightness the major explanation why unemployed workers find it easier to get a job in the services sector than in the manufacturing one.

As for the sectoral steady state ratios of the unemployment compensation over aggregate salary, $\gamma^c_{\mathfrak{m}} \approx 0.24$ and $\gamma^c_{\mathfrak{s}} \approx 0.29$, their central estimates are slightly larger to what we find as the sample average ratio of the expenses from unemployment compensation and wage bonus, released by the Brazilian Ministry of Finances,

[11] See Hosios (1990) for the intuition behind this result.

[12] See e.g. Andolfatto (1996), Blanchard and Diamond (1989), Flinn (2006), Hagedorn and Manovskii (2008), Hall (2005), Merz (1995), Mortensen and Nagypal (2007), and Shimer (2005).

over the salaries/wages components from the nominal income, released by IBGE. That is, we find an average ratio of 0.199 from 2002:Q1 on.

Table 1: Estimated Parameters

Parameter		Parameter		Parameter	
$\varsigma_{\mathfrak{m}\mathfrak{m}}$	0.08 (0.07,0.10)	ν	6.60 (5.14,7.99)	φ_y	2.44 (1.47,3.43)
$\varsigma_{\mathfrak{m}\mathfrak{s}}$	0.04 (0.03,0.06)	$\iota_{\mathfrak{c}}$	0.75 (0.63,0.89)	$\phi^{\rho}_{\mathfrak{m}}$	0.86 (0.76,0.95)
$\delta^{\mathfrak{m}}_{\mathfrak{m}}$	0.53 (0.50,0.56)	$\varepsilon_{\mathfrak{m}}$	0.98 (0.96,1.00)	$\phi^{\rho}_{\mathfrak{s}}$	0.97 (0.94,0.99)
$\delta^{\mathfrak{s}}_{\mathfrak{s}}$	0.88 (0.82,0.95)	$\varepsilon_{\mathfrak{s}}$	0.66 (0.54,0.78)	$\phi^{\eta}_{\mathfrak{m}}$	0.90 (0.82,0.99)
$\bar{\delta}^{\mathfrak{s}}_{\mathfrak{m}}$	0.91 (0.84,0.99)	$\alpha_{\mathfrak{m}}$	0.78 (0.75,0.81)	$\phi^{\eta}_{\mathfrak{s}}$	0.10 (0.00,0.24)
$\bar{\delta}^{\mathfrak{m}}_{\mathfrak{s}}$	0.08 (0.00,0.17)	$\alpha_{\mathfrak{s}}$	0.31 (0.28,0.33)	$\phi_{\mathfrak{c}}$	0.19 (0.08,0.27)
$a_{\mathfrak{m}}$	0.95 (0.92,0.99)	$\iota_{\mathfrak{m}}$	0.91 (0.81,1.00)	$\phi^{b}_{\mathfrak{m}}$	0.06 (0.00,0.12)
$a_{\mathfrak{s}}$	0.96 (0.81,0.99)	$\iota_{\mathfrak{s}}$	0.06 (0.00,0.12)	$\phi^{b}_{\mathfrak{s}}$	0.09 (0.00,0.20)
$\bar{b}_{\mathfrak{m}}$	0.95 (0.90,0.99)	$\bar{\mathfrak{p}}^{\mathsf{ue}}_{\mathfrak{m}}$	0.05 (0.00,0.10)	$\phi^{\mathsf{a}}_{\mathfrak{m}}$	0.71 (0.56,0.83)
$\bar{b}_{\mathfrak{s}}$	0.55 (0.43,0.67)	$\bar{\theta}^{e}_{\mathfrak{m}}$	0.85 (0.56,1.16)	$\phi^{\mathsf{a}}_{\mathfrak{s}}$	0.90 (0.83,0.98)
$\gamma^{c}_{\mathfrak{m}}$	0.24 (0.16,0.32)	$\bar{\theta}^{e}_{\mathfrak{s}}$	1.78 (1.55,2.00)	$\phi^{\mathfrak{m}}_{\mathfrak{s}}$	0.05 (0.00,0.09)
$\gamma^{c}_{\mathfrak{s}}$	0.29 (0.19,0.38)	φ_i	0.98 (0.97,0.99)	$\phi^{\mathfrak{s}}_{\mathfrak{s}}$	0.95 (0.91,0.99)
σ	1.85 (1.01,2.61)	φ_π	3.21 (1.52,4.73)		

T=48, N of Series: 13, point estimate: posterior mean, parentheses: 90% *HPD credible intervals*
 # Kept Draws: 125000, diagnostics: model log marginal likelihood (lml): 1632.81

Table 2: Estimated Standard Deviations

Shock	St Dev	Shock	St Dev
$\epsilon^{\rho}_{\mathfrak{m},t}$	0.037 (0.030,0.043)	$\epsilon^{b}_{\mathfrak{m},t}$	0.205 (0.008,0.455)
$\epsilon^{\rho}_{\mathfrak{s},t}$	0.734 (0.605,0.861)	$\epsilon^{b}_{\mathfrak{s},t}$	0.229 (0.124,0.332)
$\epsilon_{\ell,t}$	0.001 (0.001,0.001)	$\epsilon^{\mathsf{a}}_{\mathfrak{m},t}$	0.019 (0.016,0.021)
$\epsilon^{\eta}_{\mathfrak{m},t}$	0.022 (0.017,0.026)	$\epsilon^{\mathsf{a}}_{\mathfrak{s},t}$	0.011 (0.008,0.014)
$\epsilon^{\eta}_{\mathfrak{s},t}$	0.013 (0.010,0.016)	$\epsilon^{\mathfrak{s}}_{\mathfrak{m},t}$	8.935 (6.012,12.022)
$\epsilon_{\mathfrak{c},t}$	0.109 (0.056,0.162)	$\epsilon^{\mathfrak{s}}_{\mathfrak{s},t}$	0.171 (0.088,0.251)
		$\epsilon_{\mathrm{i},t}$	0.002 (0.002,0.003)

T=48, N of Series: 13, point estimate: posterior mean, parentheses: 90% *HPD credible intervals*
 # Kept Draws: 125000, diagnostics: model log marginal likelihood (lml): 1632.81

The data also suggests that the reciprocal of the marginal rate of intertemporal substitution is similar to what is found in the US ($\sigma \approx 1.85$),[13] and well identified. As for the reciprocal ν of the Frisch elasticity, its marginal posterior distribution suggest that there is no labor supply puzzle in the Brazilian labor market, i.e. its central estimate implies that labor is just weakly elastic $\left(\frac{1}{\nu} \approx \frac{1}{6.6} = 0.15\right)$ to salaries in Brazil. This result is in line with the international micro evidence. Indeed, Chetty et al. (2011) shows that macro evidence

[13] See e.g. Smets and Wouters (2007).

tends to mimic micro evidence when labor is split into its extensive and intensive margins in macroeconomic models. The results also suggest that workers are much more productive, on average, in the manufacturing sector than those from the services sector ($\varepsilon_{\mathfrak{m}} \approx 0.98 > \varepsilon_{\mathfrak{s}} \approx 0.66$). Moreover, prices are much stickier ($\alpha_{\mathfrak{m}} \approx 0.78 > \alpha_{\mathfrak{s}} \approx 0.31$) and much more persistent ($\iota_{\mathfrak{m}} \approx 0.91 > \iota_{\mathfrak{s}} \approx 0.06$) in the manufacturing sector than in the services sector. Since prices are more flexible in the services sector, its real side is not as much affected by monetary policy as it is in the manufacturing sector. And, even though price rigidity is stronger in the manufacturing sector, sectoral inflation dynamics must not detach as much due to strategic complementarity.

4 Impulse responses

Figures 9 to 13 show the impulse responses to a monetary policy shock (1 p.p. annualized), for the heterogeneous model. Note that, as expected, aggregate inflation, GDP, real salary, real wages and employment fall, while the unemployment rate increases. However, the reduction of real salaries and wages induces workers that were previously out of the labor market to return as unemployed workers, and hence the participation rate increases. This fact, in turn, induces the unemployment rate to rise more than 1 to 1 than the fall in employment. Note that this model is able to capture what was know as labor hoarding. After the shock, both sectors tend to reduce hours (intensive margin of labor) as the shock heats, but employment takes longer to fall.

As for the sectoral responses, the rise in the participation rate is mostly observed in the services sector. This is due to the fact that the unemployment duration is 10 times as large in the manufacturing sector as it is in the services sector, after a monetary shock. Moreover, real wages and salaries is not as much affected in the Brazilian services sector. Since as more workers become unemployed in the manufacturing sector and they find it easier to reallocate to the services sector, they tend to reallocate after a small training period out of the labor market. Therefore, the participation rate in the manufacturing sector actually falls.

The response of the services output (not GDP) to a monetary policy shock has mixed dynamics. Since prices are more flexible in this sector, its real side is less sensible to monetary policy changes. Therefore, in the short run, output in the services sector tends to rise due to the fact that wages and salaries are falling and the availability of unemployed workers in increasing in this sector. In a nutshell, producing becomes cheaper,[14] and so the sector can produce more. In the medium run (5 to 20 quarters), the fall in demand induces this sector to reduce production. The manufacturing sector, on the other hand, experience a much faster and stronger reaction to the monetary shock, falling 20 times as stronger than in the services sector.

As for sectoral GDP, we must discount intermediate consumption from output in order to obtain value added. In this model, the only intermediate consumption is what firms consumes in order to post vacancies. Since the services sector faces an increasing supply of unemployed workers, its firms find it optimal to reduce

[14] Note that this results may not arise in models in which firms need to borrow to finance its inputs allocations.

even further their expenses in vacancy postings. Therefore, services GDP tends to rise above what services GDP does in the short run.

5 Conclusion

This paper presents and estimate a novel way to model the labor and goods markets with heterogeneous sectors in Brazil, endogenizing the optimal decision to reallocate to another sector or leave the labor market.

The major empirical findings are that: (i) workers from the manufacturing sector who are out of the labor market take longer to return (1.9 quarters) than workers from the service sector (1.1 quarters) ; (ii) workers from the manufacturing sector reallocate much faster to the service sector (2.3 quarters) than workers from the services sector - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare; (iii) it is the labor market tightness the major explanation why unemployed workers find it easier to get a job in the services sector than in the manufacturing one; (iv) although unemployed workers from the service sector find it easier to get a job than workers from the manufacturing sector, the workers' bargaining power in the manufacturing sector is much larger than the bargaining power in the service sector. As a result, the average salary in the service sector are more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The results also suggest that salary bargaining is much more efficient in the manufacturing sector. The data also support the evidence that there is no labor supply puzzle in the Brazilian labor market, i.e. I find that labor is just weakly elastic to salaries in Brazil.

The results also suggest that workers are much more productive, on average, in the manufacturing sector than those from the services sector. Moreover, prices are much stickier and much more persistent in the manufacturing sector than in the services sector. Since prices are more flexible in the services sector, its real side is not as much affected by monetary policy as it is in the manufacturing sector. And, even though price rigidity is stronger in the manufacturing sector, sectoral inflation dynamics must not detach as much due to strategic complementarity. As for the dynamics after a monetary policy shock, the results imply that it is the manufacturing sector which suffers more. The fall in employment, hours, real salaries, GDP and output is much stronger in the manufacturing than in the services sector. The model is also able to capture what is know as labor hoarding, for hours tend to fall much faster than employment after the shock.

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A Figures

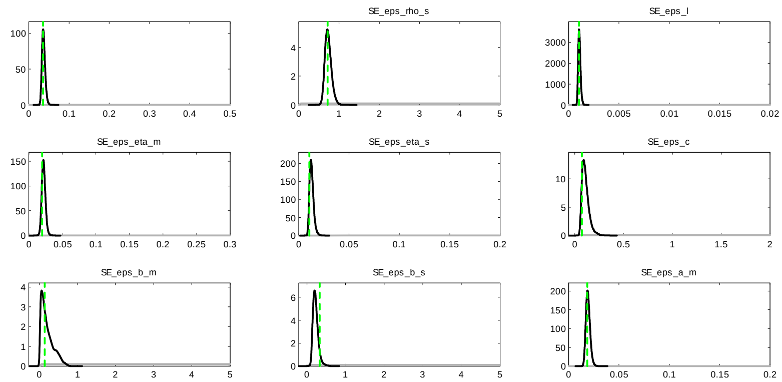
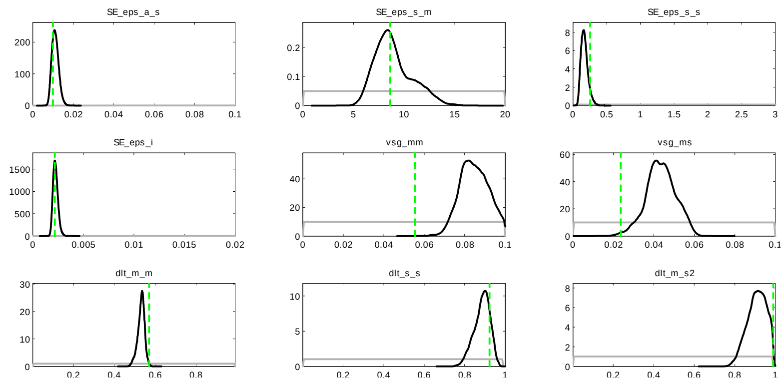


Figure 3: Posterior Marginal Distributions



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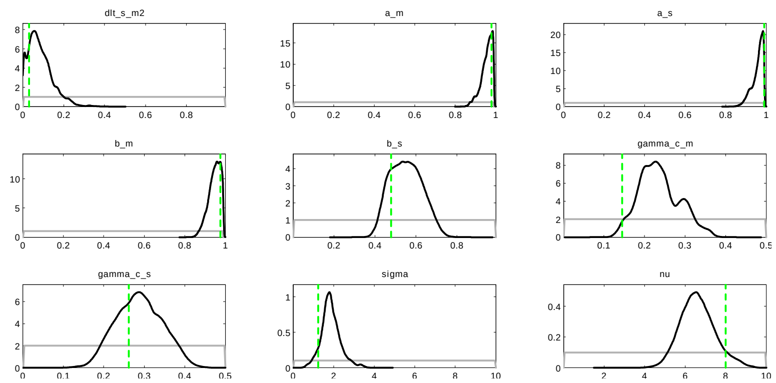

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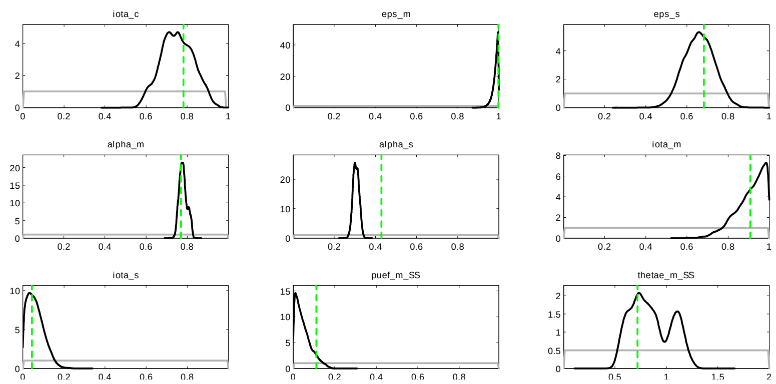

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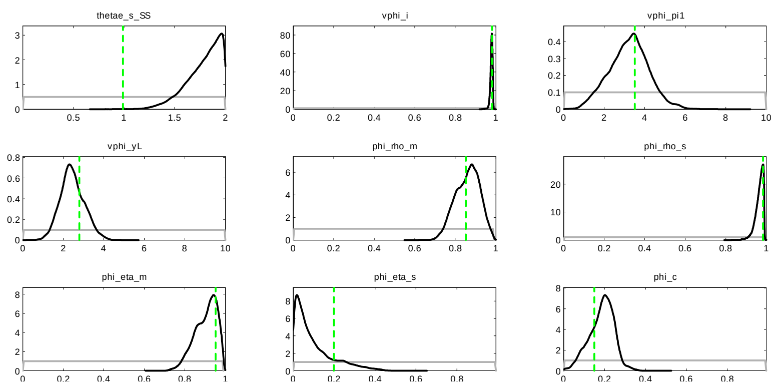

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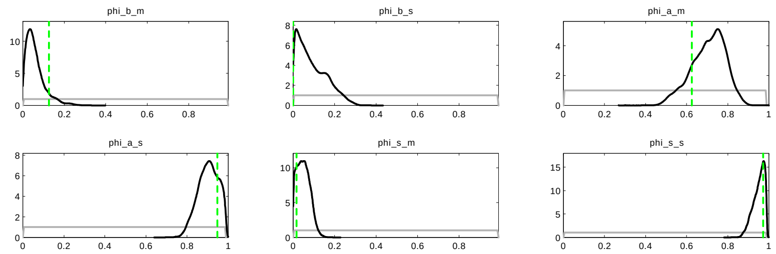

Figure 8: Posterior Marginal Distributions

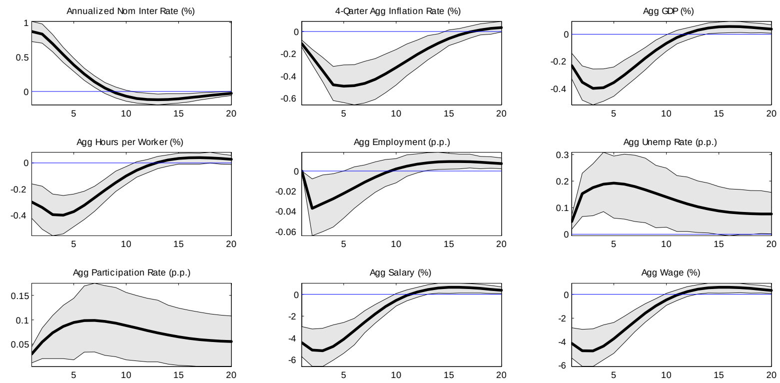

Figure 9: Impulse Responses to a 1 p.p. Annualized Interest Rate

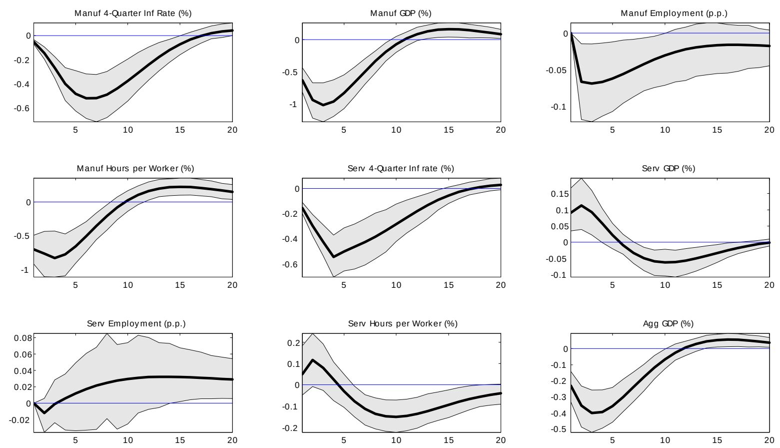

Figure 10: Impulse Responses to a 1 p.p. Annualized Interest Rate

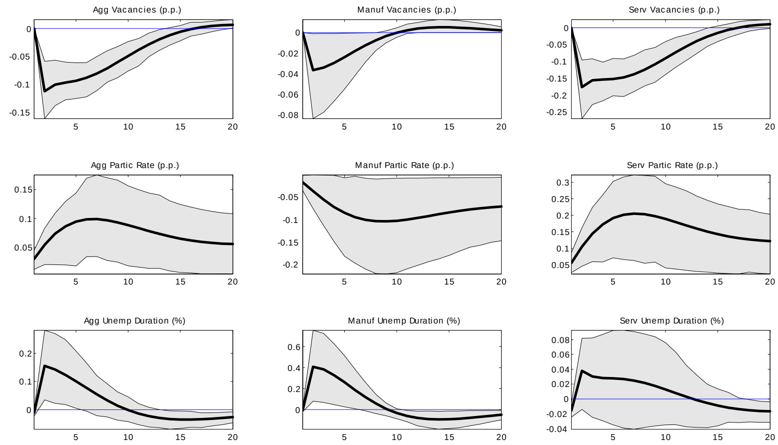

Figure 11: Impulse Responses to a 1 p.p. Annualized Interest Rate

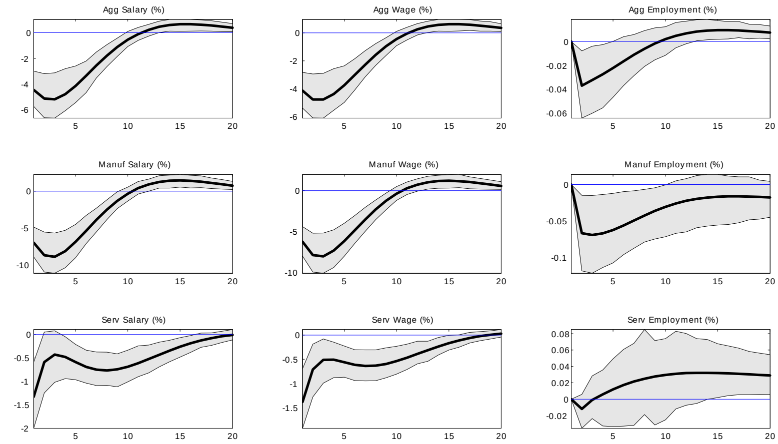

Figure 12: Impulse Responses to a 1 p.p. Annualized Interest Rate

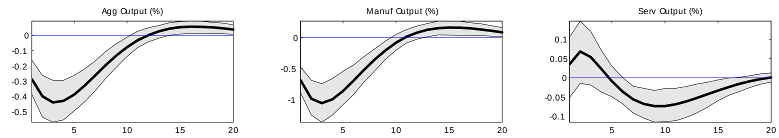

Figure 13: Impulse Responses to a 1 p.p. Annualized Interest Rate